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Vibration Analysis of Non-Linear Beam Subjected to a Moving Load by Using the Galerkin Method

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TABLE OF CONTENTS

NOMENCLATURE

u :	Longitudinal deflection	(m)
w :	Transverse deflection	(m)
ρ	Mass per unit volume	(kg/m^3)
E	Young's modulus	(N/m^2)
A	Cross-sectional area	(m^2)
I	Moment of inertia of area	(m^4)
m_s	Body mass of the vehical	(Kg)
m_u	Mass of the wheels	(Kg)
g	Gravitational acceleration	(m/s^2)
k_s	Suspension stiffness	(N/m)
k_p	Tire stiffness	(N/m)
v	Velocity of the moving load	(m/s)
C_s	Damping coefficient of the moving load	(Ns/m)
C	Damping coefficient of the beam material	(Ns/m)
t	Time	(s)
x	Distance from the origin	(m)
y_1	Deflection of the beam at the point just under the moving load	(m)
r	Radius of gyration of the cross section	
y_2	Vertical displacement of the wheels	(m)
y_3	Vertical displacements of the vehicle body	(m)
Q_s	Load parameter	
$\dot{y}_2$	First derivative of y_2 with respect to t	
$\dot{y}_3$	First derivative of y_3 with respect to t	
y	Vertical coordinate measured from the origin	

LIST OF FIGURES

LIST OF APPENDICES

ABSTRACT

Vibration Analysis of a Non-Linear Beam Subjected to a Moving Load by Using the Galerkin Method

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Supervised By

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In designing the highway bridges to be subjected to moving traffic loads, it is desirable to avoid excessive dynamic deflections, because of additional safety considerations, and possible discomfort of pedestrians and vehicle drives. Therefore, analytical or numerical methods of predicting the dynamic deflections are needed in designing bridges; to ensure that these will be acceptable.

This study presents the analysis of dynamic deflections of a beam (with the bridge modeled as a beam), including the effects of geometric non-linearity, subjected to moving vehicle load. The beam is assumed to be elastic and simply supported with immovable ends, and the vehicle is assumed to be a two degree of freedom, with the vehicle moving on the beam from one end to another. The dynamic deflections of the beam and vehicle are computed by using the Galerkin method, by which these deflections are assumed to be a set of time functions multiplied by assumed approximate space functions. The time functions are numerically computed by solving the non-linear differential equations by the Newmark-β method.

CHAPTER ONE

INTRODUC TION AND LITERATURE REVIEW

1.1 INTRODUCTION:-

It is a well-known fact that beams represent one of the most important structural members in engineering design and construction and there is no design in which the beam problems in one form or another do not arise.

In the analysis of large amplitude deflections in beam under lateral loads one cannot neglect the non-linear effects in the equations of motion. Particularly for relatively slender beams whose edges are restrained from axial displacement, the deflections calculated for linear and non-linear beams show large differences and hence the analysis of deflections of non-linear beam is necessary in practice.

Non-linearities in the behavior of a structure are due primarily to either of the two causes as proposed by [1]. 1. The most obvious cause is a material having a nonlinear stress-strain curve; in this case we refer to the structure as having material nonlinearities. 2. The other possibility is that the nonlinearities are produced by the geometry of the deflected structure. This situation occurs whenever the deflections of the structure alter the action of the applied loads or the reactions. In our case it is due to the axial force generated by stretching the middle surface due to the immovability of the end support.

Three Chapters are devoted for obtaining the non-linear transverse deflection. Chapter 2 is devoted to the mathematical modeling of the problem and the selection of the approximation functions that meet the specified boundary conditions . The technique of obtaining algebric equations in terms of the undetermined parameters are also discused in Chapter 2. Chapter 3 is devoted to the study of the method of solution as applied to the final differential equations which is obtained from Chapter 2. Numerical example and the discussion of the results are presented in Chapter 4.

1.2 ILLUSTRATION OF THE PRESENT WORK

It is a well-known fact that the moving loads on the beam produce greater dynamic deflections and greater stress than do the same loads acting statically and because one cannot neglect the non-linear effects in the equation of motion. Therefore the objectives of this study are :

1. To find the dynamic deflection of a beam subjected to a moving load by using the Galerkin method, which is a form of the method of weighted residuals (MWR) [2,3], including the effects of geometric non-linearity.
2. To investigate the effects of damping on the response.
3. To investigate the changes in response due to variations in the vehicle velocity and in the vehicle load.

1.3 LITERATURE SURVEY

A variety of studies of non-linear beams have been made, for both free and steady state vibration problem [4-10], by using a finite element approach [4-8] and a continuum approach [9,10], (Perturbation and Rayleigh-Ritz methods) . These studing indicate that the deflections which were calculated for linear and non-linear beams show large differences.

As stated earlier, moving loads on the beam may produce a larger dynamic deflection and thus a greater stress than do the same loads acting statically, and a number of investigation of the vibrations of beams subjected to a moving loads have been reported [11-24]. Even though there are many published reports and studies on this subject, this survey will cover the recent studies done on this topic.

Bridges are systems which possess an infinite number of degrees of freedom because the mass and elasticity are continuously distributed. Various investigations of the dynamic behavior of such continuous systems subjected to a moving load have been carried out by [11-18] for deterministic excitation, and by [19-21] for random excitation.

L.Fryba [19], investigated the non-stationary random vibrations of a beam. The beam was subjected to a random force with constant mean value moving with constant speed along the beam.The statistical characteristics of the first and second order for the deflection was computed by using the correlation method. The numerical results of the coefficient of variation of the deflection of the beam span mid-point were given for five basic types of convariances of the force (white noise, constant, exponential cosine, exponential and cosine wave). The effect of the speed of the movement of the force along the beam as well as the effect of the beam damping was investigated in detail.

Bridges encountered in practice frequently have appreciable non-uniformity of cross sectional area and moment of area. J.Hino [22], described the evaluation of the dynamic deflections and accelerations of a concrete bridge. The bridge was assumed to be completely elastic and to be subjected to a moving traiffic load. Both the cross sectional area and the moment of area being non-uniform. The vehicle was assumed to be a single degree of freedom system, with mass, stiffness and damping. The method of weighted residuals (MWR) in the Galerkin form was used to formulate the finite element problem, then Wilson's H method [25], which is one of the direct integral methods, was used for integrating the differential terms of the final formulation of the spatial domain. In the above investigation the effects of longitudinal deflections and inertia were neglected. However, there exist cases in which the longitudinal deflections and inertia cannot be neglected. Hino *et al.* [23], presented the non-linear vibration of immovably supported variable beams, in which the geometric non-linearity due to the axial force generated by stretching the middle surface was taken into account. When the beam is subjected to a moving load, the dynamic deflection of the beam were computed by using the Galerkin finite element formulation, and the time differential terms were integrated by using the implicit direct integrating method. Non-linear deflections of the beam have been calculated for the following models. In the first model, the longitudinal deflections and inertia were considered; model II, the longitudinal deflections were considered,but the longitudinal inertia was not considered: i.e., and then $EA\left[\left(\partial u/\partial x\right) + \frac{1}{2}\left(\partial w/\partial x\right)^2\right]$=constant ; model III, the longitudinal effects were not considered. [9,26]: i.e., $\rho A\left(\partial^2 u/\partial t^2\right) = 0$ and $\left(\partial u/\partial x\right) = 0$. The results obtained by using these non-linear models were numerically compared with that obtained by the linear model . In the above investigation the effects of the damping coefficient of the beam material were neglected from the governing differential equation.

Yoshimaura *et al* [24] , investigated the dynamic deflection of a beam, including the effects of geometric non-linearity and the damping coefficient of the beam material, subjected to moving vehicle loads, where the vehicle was assumed to be a single degree-of-freedom. The dynamic deflections of the beam and vehicles were computed by using the Galerkin method. The dynamic deflections were assumed to be a set of time functions multiplied by approximate functions, respectively, and the time functions were numerically computed by solving the non-linear differential equations by the Newmark- β method. This latter method will be discussed later in chapter 3.2.

CHAPTER TWO

MATHEMATICAL MODELLING

2.1 GENERAL

Virtually every phenomenon in nature, whether biological, geological, or mechanical, can be described with the aid of the laws of physics, in terms of algebraic, differential, or integral equations relating various quantities of interest. Determining the stress distribution in a pressure vessel with oddly shaped holes and numerous stiffeners and subjected to mechanical, thermal, and/or aerodynamic loads, and finding the concentration of pollutants in sea water or in the atmosphere, and simulating weather in an attempt to understand and predict the mechanics of formation of tornadoes and thunderstorms are a few examples of many important practical problems. While derivation of the governing equations for these problem is not unduly difficult, their solution by exact method of analysis is a formidable task. In such cases approximate method of analysis provide alternative means of finding solutions. Among these the variational methods such as the Ritz and Galerkin methods.

In this Chapter we will begin by presenting and discussing the structure of the problem , such as the equations of motion which will be presented in section 2.2 , and then we will present in section 2.3 the procedure to get a set of ordinary differential equations in time by using the MWR.

2.2 MATHEMATICAL MODELLING

The system under consideration is shown in Figure 1. The vehicle is modelled as a two-degrees of freedom and is moving with constant velocity on the beam from one end to the other. The dynamic deflection of the moving vehicle, which is always in

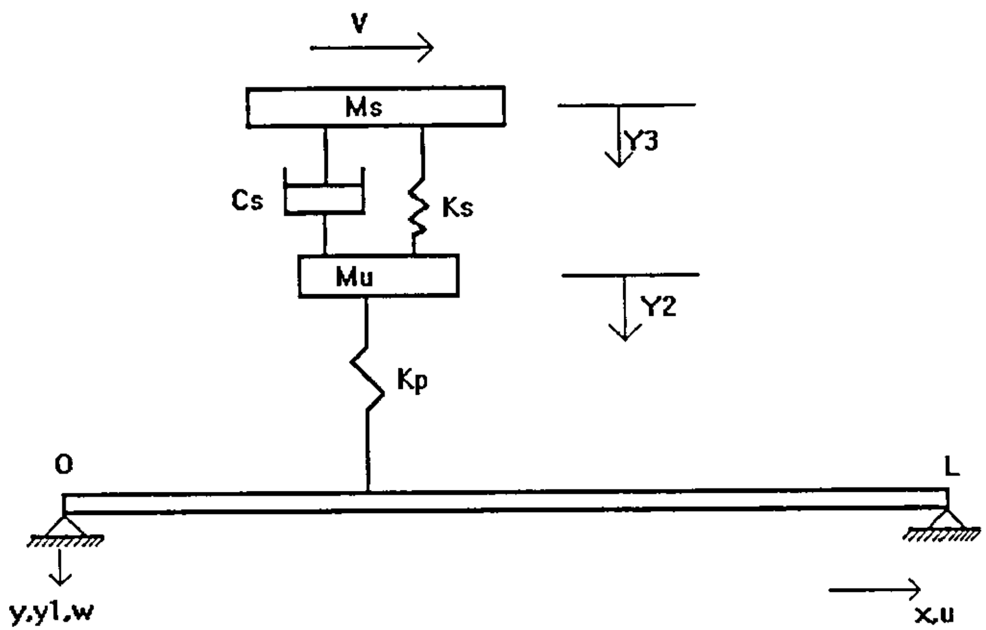

Figure1 . Model of beam and moving load [23].

contact with the beam, is expressed by y_2 and y_3 , with the origin (with O as the origin) assumed to be the neutral position.

This model differs from other models (which were used by investigators mentioned in the literature survey) by the following points:

1. The moving load is modelled as a two degrees of freedom, while most of the investigators assumed the moving vehicle to be a single degree-of-freedom.

2. In this model the effects of the damping coefficient of the beam material will be included in the calculations . This factor was neglected by many investigators.

3. The effects of the longitudinal deflections will be considered in the equations of motion. The above term was neglected by many investigators.

Literature survey showed that the same model was used by Hino *et al* [23] , with the exception of point 2 and he solved the problem by the finite element formulation.

When the rotary inertia and the shearing deformations are neglected (because the beam is assumed to be thin and also these terms are small and they are usually ignored compared with the transverse deflection.), the governing differential equations (see Appendix I for the derivations) for the non-linear flexural deflections of the beam (including the geometric non-linearity) [1,23] , are

$$\rho A(\partial^2 w/\partial t^2)+C(\partial w/\partial t)+(\partial^2/\partial x^2)(EI \quad \partial^2 w/\partial x^2)=$$
$$(\partial/\partial x)\left\{EA\left[(\partial u/\partial x)+\frac{1}{2}(\partial w/\partial x)^2\right]\partial w/\partial x\right\}+[(m_s+m_u)g-m_s\ddot{y}_3-m_u\ddot{y}_2]\delta(x-vt), \qquad (1).$$

where

$(m_u+m_s)g$: is the static load.

$m_s\ddot{y}_3+m_u\ddot{y}_2$: is the dynamic load.

For the axial deflection, we have

$$\rho A(\partial^2 u/\partial t^2)-(\partial/\partial x)\left\{EA\left[(\partial u/\partial x)+\frac{1}{2}(\partial w/\partial x)^2\right]\right\}=0, \qquad (2).$$

The vehicle motion is governed by the following equations

$$m_u(d^2y_2/dt^2)+m_s(d^2y_3/dt^2)+k_p(y_2-y_1)=0, \qquad (3).$$

$$m_s(d^2y_3/dt^2)+c_s(dy_3/dt-dy_2/dt)+k_s(y_3-y_2)=0, \qquad (4).$$

where

$y_1 = w$ at $x = vt$

The boundary conditions for the simply supported beam with immovable ends are given by :

$u = 0.$ at $x = 0.$ and $x = L$

$w = 0.$ and $EI(\partial^2 w/\partial x^2) = 0$ at $x = 0.$ and $x=L$ (5).

and the initial conditions for the beam and for the moving vehicle at $t = 0.0$ are, respectively,given by:

$$u = u_0(x), \quad \partial u/\partial t = \dot{u}(x), \quad w = w_0(x), \quad \partial w/\partial t = \dot{w}(x), \qquad (6).$$

$$y_2 = y_{2_0}, \quad dy_2/dt = \dot{y}_{2_0}, \quad y_3 = y_{3_0}, \quad dy_3/dt = \dot{y}_{3_0}, \qquad (7).$$

Equation (1) can be expressed by different way , this is done by rearranging the expression in equation (3) as :

$$m_s(d^2y_3/dt^2)+m_u(d^2y_2/dt^2)=-k_p(y_2-y_1)$$

By substituting the above equation into equation (1) we will have the following equation:

$$\rho A(\partial^2 w/dt^2)+C(\partial w/\partial t)+(\partial^2/\partial x^2)[EI\,\partial^2 w/\partial x^2]=$$
$$(\partial/\partial x)\{EA[(\partial u/\partial x)+1/2(\partial w/\partial x)^2]\partial w/\partial x\}+[(m_s+m_u)g+k_p(y_2-y_1)]\delta(x-vt)$$

Here x is the axial co-ordinate measured from the origin (m) (with O as the origin), u is the longitudinal displacement (m), w is the transverse deflection (m) , ρ is the mass per unit volume (kg/m^3), E is Young's modulus (N/m^2), A is the cross-sectional area (m^2) , I is moment of inertia of area (m^4), m_s denotes the body mass of the vehicle, (kg), m_u the mass of the wheels (Kg), g is gravitational acceleration (m/s^2), k_p denotes the tire stiffness and k_s the suspension stiffness constants of the moving load (N/m), C_S is the damping coefficient of the moving load (Ns/m), y_3 denotes the vertical displacement of the body (m), y_2 is the vertical displacement of the wheels (m), y_1 is the defle… of the beam at the point just under the moving load (m), v is the velocity of the … load (m/s), $\delta(.)$ is Dirac's delta function , C is the damping coefficient of the … m material , t is time (s), $\dot{y}_2$ and $\dot{y}_3$ are the first derivative of y_2 and y_3 w… t (m/s).

… generalize the results equations (1)-(4) must be written in non … s and this is done by using the following dimensionless variables [23] … ess variables are denoted by astreisks).

$$…/\rho)t, \quad v^* = \sqrt{(\rho/E)}v, \quad x^* = x/L, \quad A^* = A/L^2,$$

$$…\sqrt{E/\rho}, \quad I^* = I/L^4, \quad y_3^* = y_3/L, \quad y_2^* = y_2/L,$$

$$…,(r = \sqrt{I/A}), \quad g^* = L\rho g/E, \quad \zeta_n = m_u/(m_s + m_u),$$

$$…_v = (m_s + m_u)/L^3\rho, \quad \xi = \sqrt{k_s L^2 \rho / E(m_s + m_u)},$$

$$\hbar = C_S / 2\sqrt{K_S(M_S + M_U)}, \quad \lambda = \sqrt{K_P L^2 \rho/(M_S + M_U)},$$

$$w^* = w/L, \quad u^* = u/L, \quad L^* = L/L,$$

Here r is the radius of gyration of the cross section. By substituting these dimensionless quantities into equations (1)-(4), and dropping the asterisks for brevity, gives

$$A(\partial^2 w/\partial t^2) + C(\partial w/\partial t) + (\partial^2/\partial x^2)[I(\partial^2 w/\partial x^2)] = (\partial/\partial x)\left\{A\left[(\partial u/\partial x) + \frac{1}{2}(\partial w/\partial x)^2\right]\partial w/\partial x\right\} + m_v[g - (1-\zeta_n)\ddot{y}_3 - \zeta_n \ddot{y}_2]\delta(x - vt), \quad (8)$$

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$$A(\partial^2 u/\partial t^2) - (\partial/\partial x)[A(\partial u/\partial x)] = \frac{1}{2}(\partial/\partial x)\left[A(\partial w/\partial x)^2\right], \quad (9).$$

$$\zeta_n(d^2 y_2/dt^2) + (1-\zeta_n)(d^2 y_3/dt^2) + \lambda^2(y_2 - y_1) = 0.0, \quad (10).$$

$$(1-\zeta_n)(d^2 y_3/dt^2) + 2\hbar\xi(dy_3/dt - dy_2/dt) + \xi^2(y_3 - y_2) = 0.0, \quad (11)$$

2.3 APPLICATION OF THE MWR TO THE SYSTEM

Our objective in this section is to apply the variational method of Weighted Residuals to the solution of equations (8)-(11). This method ,like other variational methods such as Ritz method ,Least-squares method, collocation method and Courant method, seek an approximate solution in the form of a linear combination of suitable approximation function.These methods differ from each other in the choice of the approximate functions.The above methods provide simple means of finding approximate solutions to physical problems . The formative and computational efforts involved are less compared to most other methods, such as the finite element method (The FEM is the most flexible teqnic available for handling problem with complicated B.C, but requires fine model to represses the motion . This places large requirements on the size of the matrices and consequently large computing time. However since the B.C of this problem is relatively simple ,one can use the variational method such as the Galerkin method. Which has the advantage that the motion of the system can be represented by small number of functions which reduces the size of the working matrices and in addition this problem has been solved by the FEM).

The procedure of the method of weighted-residual consists of the following steppes :

1. Considering the operator equation

$$Aw = F \quad \text{in} \quad \Omega \qquad \text{I}$$

where A is an operator (linear or nonlinear), often a differential operator , acting on the unknown dependent variable w , and F is known function of position . The function w (i.e., solution) is not only required to satisfy the operator equation (I) ,it is also required to satisfy the boundary conditions associated with the operator.

2. Assume an approximate solution:

In the MAR the solution w is approximated by expression of the form

$$w_n = \phi_0 + \sum_{j=1}^{N} C_j \phi_j \qquad \text{II}$$

the requirements of ϕ_0 C_j and ϕ_j will be noted shortly.

3. Substitution of the approximation (II) into the operator equation (I) results in a residual (i.e., an error in the equation)

$$E \equiv A(w_n) - f \neq 0. \qquad \text{III}$$

Once the ϕ_0 and ϕ_j are selected , E is merely a function of the independent variables and the parameters C_j . In the MAR the parameters are determined by setting the integral (over the domain) of a weighted residual of the approximation to zero:

$$\int_{\Omega} \psi_i(x,y) E(x,y,C_j)\, dxdy = 0. \qquad i=1,2,\dots\dots,N \qquad \text{IV}$$

where ψ_j are weight functions.

When the MWR (Method of Weighted residuals) is applied to the derivation of the solution of the above equations (8)-(11), the solution for w(x,t) and u(x,t) are approximated by the following expressions.

$$\tilde{w}(x,t) = \alpha_0(x) + \sum_{i=1}^{N_1} \alpha_i(x)\theta_i(t), \qquad (12),$$

$$\tilde{u}(x,t) = \phi_0(x) + \sum_{k=1}^{N_2} \phi_k(x)\psi_k(t), \qquad (13),$$

Where the $\alpha_i(x)$'s and $\phi_i(x)$'s are,respectively,sets of approximating functions which must satisfy the following conditions:

1- $\alpha_i(x)$ and $\phi_i(x)$ must satisfy at least the homogeneous form of the essential boundary conditions of the problem.(It follows that the specification of u and w in equation (5) constitutes the essential boundary conditions and the specification of $EI\frac{\partial^2 w}{\partial x^2}$ constitutes the natural boundary conditions).

2. For any $N_1 \& N_2$, the sets $\{\alpha_i(x)\}_{i=1}^{N_1}$ and $\{\phi_k(x)\}_{k=1}^{N_2}$ are linearly independent.

3. $\{\alpha_i(x)\}$ and $\{\phi_k(x)\}$ are complete.

The functions $\alpha_0(x)$ and $\phi_0(x)$ must satisfy all the specified boundary conditions ($\alpha_0(x)$ and $\phi_0(x) = 0$ if all the specified boundary conditions are homogeneous,i.e.,if the value of $u_0(x), w_0(x), \dot{u}_0(x)$ and $\dot{w}_0(x) = 0$ for the beam and $y_{2_0}, \dot{y}_{2_0}, \dot{y}_{3_0}$ and $y_{3_0} = 0$ for the moving vehicle) of the problem. The $\theta_i(t)$'s and $\psi_i(t)$'s are, respectively, a set of time functions which are to be determined and N_1 and N_2 correspond to the numbers of terms necessary in the approximations. When equations (12) and (13) are substituted into equations (6),(8) and (9), the residuals (which are defined as the difference between the approximate and exact solutions ,i.e., an error in the equations) are , respectively, given by

$$\varepsilon_1 = A(\partial^2\tilde{w}/\partial t^2) + C(\partial\tilde{w}/\partial t) + (\partial^2/\partial x^2)\left[I(\partial^2\tilde{w}/\partial x^2)\right] -$$
$$(\partial/\partial x)\left\{A\left[(\partial\tilde{u}/\partial x) + \frac{1}{2}(\partial\tilde{w}/\partial x)^2\right](\partial\tilde{w}/\partial x)\right\} - m_v\left[g - (1-\zeta_n)\ddot{y}_3 - \zeta_n\ddot{y}_2\right]\delta(x - vt) \quad (14)$$

$$\varepsilon_2 = A(\partial^2\tilde{u}/\partial t^2) - (\partial/\partial x)\left[A\left\{(\partial\tilde{u}/\partial x) + \frac{1}{2}(\partial\tilde{w}/\partial x)^2\right\}\right], \quad (15).$$

$$\varepsilon_{11} = \tilde{w}_0(x) - w_0(x), \qquad \varepsilon_{12} = \dot{\tilde{w}}_0(x) - \dot{w}_0(x), \quad (16).$$

$$\varepsilon_{21} = \tilde{u}_0(x) - u_0(x), \qquad \varepsilon_{22} = \dot{\tilde{u}}_0(x) - \dot{u}_0(x), \quad (17).$$

The residuals (14) and (16) are distributed over the domain by using weighting functions (which in general,are not the same as the approximation function $\alpha_i(x)$) $\gamma_j(x)$'s , and are set to zero such that

$$\int_0^L \varepsilon_1\gamma_j(x)dx = 0.0, \qquad j = 1,2,3,.........,N_1, \quad (18).$$

$$\int_0^1 \varepsilon_{11}\gamma_j(x)dx = 0.0, \qquad \int_0^L \varepsilon_{12}\gamma_j(x)dx = 0.0, \quad (19).$$

Similarly, for the residuals (15) and (17), weighting functions $\lambda_s(x)$'s are used such as

$$\int_0^L \varepsilon_2\lambda_s(x)dx = 0.0 \qquad s=1,2,...............N_2, \quad (20).$$

$$\int_0^L \varepsilon_{21}\lambda_s(x)dx = 0.0 \qquad \int_0^1 \varepsilon_{22}\lambda_s(x)dx = 0.0 \quad (21).$$

(m), y_1 is the deflection of the beam at the point just under the moving load (m), v is the velocity of the moving load (m/s), $\delta(.)$ is Dirac's delta function , C is the damping coefficient of the beam material , t is time (s), $\dot{y}_2$ and $\dot{y}_3$ are the first derivative of y_2 and y_3 with respect to t (m/s).

In order to generalize the results equations (1)-(4) must be written in non dimensional forms and this is done by using the following dimensionless variables [23] (The dimensionless variables are denoted by astreisks).

$$t^* = (1/L)\sqrt{(E/\rho)}t, \quad v^* = \sqrt{(\rho/E)}v, \quad x^* = x/L, \quad A^* = A/L^2,$$
$$C^* = C/L^2\sqrt{E/\rho}, \quad I^* = I/L^4, \quad y_3^* = y_3/L, \quad y_2^* = y_2/L,$$
$$r^* = r/L, \left(r = \sqrt{I/A}\right), \quad g^* = L\rho g/E, \quad \zeta_n = m_u/(m_s + m_u),$$
$$m^*_v = (m_s + m_u)/L^3\rho, \quad \xi = \sqrt{k_s L^2 \rho / E(m_s + m_u)},$$
$$\hbar = C_S / 2\sqrt{K_S(M_S + M_U)}, \quad \lambda = \sqrt{K_P L^2 \rho / (M_S + M_U)},$$
$$w^* = {}^{w}/_{L}, \quad u^* = {}^{u}/_{L}, \quad L^* = {}^{L}/_{L},$$

Here r is the radius of gyration of the cross section. By substituting these dimensionless quantities into equations (1)-(4), and dropping the asterisks for brevity, gives

$$A\left(\partial^2 w/\partial t^2\right) + C(\partial w/\partial t) + \left(\partial^2/\partial x^2\right)\left[I\left(\partial^2 w/\partial x^2\right)\right] =$$
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$$w_n = \phi_0 + \sum_{j=1}^{N} C_j \phi_j \qquad \text{II}$$

the requirements of ϕ_0 C_j and ϕ_j will be noted shortly.

3. Substitution of the approximation (II) into the operator equation (I) results in a residual (i.e., an error in the equation)

$$E \equiv A(w_n) - f \neq 0. \qquad \text{III}$$

Once the ϕ_0 and ϕ_j are selected , E is merely a function of the independent variables and the parameters C_j . In the MAR the parameters are determined by setting the integral (over the domain) of a weighted residual of the approximation to zero:

$$\int_{\Omega} \psi_i(x,y) E(x,y,C_j)\, dx dy = 0. \qquad i=1,2,\ldots\ldots,N \qquad \text{IV}$$

where ψ_j are weight functions.

When the MWR (Method of Weighted residuals) is applied to the derivation of the solution of the above equations (8)-(11), the solution for w(x,t) and u(x,t) are approximated by the following expressions.

$$\tilde{w}(x,t) = \alpha_0(x) + \sum_{i=1}^{N_1} \alpha_i(x)\theta_i(t), \qquad (12),$$

$$\tilde{u}(x,t) = \phi_0(x) + \sum_{k=1}^{N_2} \phi_k(x)\psi_k(t), \qquad (13),$$

Where the $\alpha_i(x)$'s and $\phi_i(x)$'s are,respectively,sets of approximating functions which must satisfy the following conditions:

1- $\alpha_i(x)$ and $\phi_i(x)$ must satisfy at least the homogeneous form of the essential boundary conditions of the problem.(It follows that the specification of u and w in equation (5) constitutes the essential boundary conditions and the specification of $EI\frac{\partial^2 w}{\partial x^2}$ constitutes the natural boundary conditions).

2. For any $N_1 \& N_2$, the sets $\{\alpha_i(x)\}_{i=1}^{N_1}$ and $\{\phi_k(x)\}_{k=1}^{N_2}$ are linearly independent.

3. $\{\alpha_i(x)\}$ and $\{\phi_k(x)\}$ are complete.

The functions $\alpha_0(x)$ and $\phi_0(x)$ must satisfy all the specified boundary conditions ($\alpha_0(x)$ and $\phi_0(x) = 0$ if all the specified boundary conditions are homogeneous,i.e.,if the value of $u_0(x), w_0(x), \dot{u}_0(x)$ and $\dot{w}_0(x) = 0$ for the beam and $y_{2_0}, \dot{y}_{2_0}, \dot{y}_{3_0}$ and $y_{3_0} = 0$ for the moving vehicle) of the problem. The $\theta_i(t)$'s and $\psi_i(t)$'s are, respectively, a set of time functions which are to be determined and N_1 and N_2 correspond to the numbers of terms necessary in the approximations. When equations (12) and (13) are substituted into equations (6),(8) and (9), the residuals (which are defined as the difference between the approximate and exact solutions ,i.e., an error in the equations) are , respectively, given by

$$\varepsilon_1 = A(\partial^2\tilde{w}/\partial t^2) + C(\partial\tilde{w}/\partial t) + (\partial^2/\partial x^2)\left[I(\partial^2\tilde{w}/\partial x^2)\right] -$$
$$(\partial/\partial x)\left\{A\left[(\partial\tilde{u}/\partial x) + \frac{1}{2}(\partial\tilde{w}/\partial x)^2\right](\partial\tilde{w}/\partial x)\right\} - m_v\left[g - (1-\zeta_n)\ddot{y}_3 - \zeta_n\ddot{y}_2\right]\delta(x - vt) \quad (14)$$

$$\varepsilon_2 = A(\partial^2\tilde{u}/\partial t^2) - (\partial/\partial x)\left[A\left\{(\partial\tilde{u}/\partial x) + \frac{1}{2}(\partial\tilde{w}/\partial x)^2\right\}\right], \quad (15).$$

$$\varepsilon_{11} = \tilde{w}_0(x) - w_0(x), \qquad \varepsilon_{12} = \dot{\tilde{w}}_0(x) - \dot{w}_0(x), \quad (16).$$

$$\varepsilon_{21} = \tilde{u}_0(x) - u_0(x), \qquad \varepsilon_{22} = \dot{\tilde{u}}_0(x) - \dot{u}_0(x), \quad (17).$$

The residuals (14) and (16) are distributed over the domain by using weighting functions (which in general,are not the same as the approximation function $\alpha_i(x)$) $\gamma_j(x)$'s , and are set to zero such that

$$\int_0^L \varepsilon_1\gamma_j(x)dx = 0.0, \qquad j = 1,2,3,.........,N_1, \quad (18).$$

$$\int_0^l \varepsilon_{11}\gamma_j(x)dx = 0.0, \qquad \int_0^L \varepsilon_{12}\gamma_j(x)dx = 0.0, \quad (19).$$

Similarly, for the residuals (15) and (17), weighting functions $\lambda_s(x)$'s are used such as

$$\int_0^L \varepsilon_2\lambda_s(x)dx = 0.0 \qquad s=1,2,...............N_2, \quad (20).$$

$$\int_0^L \varepsilon_{21}\lambda_s(x)dx = 0.0 \qquad \int_0^l \varepsilon_{22}\lambda_s(x)dx = 0.0 \quad (21).$$

By using equations (18) and (20), the unknown time functions, the $\theta_i(t)$'s and $\psi_k(t)$'s , are , respectively, determined, depending upon the selected functions, $\gamma_j(x)$,s and $\lambda_s(x)$,s. The Galerkin method selection ,which is the most widely used, is to choose $\gamma_j(x)$'s and $\lambda_s(x)$'s as

$$\gamma_j(x) = \alpha_j(x), \qquad j = 1,2,\ldots\ldots\ldots\ldots, N_1, \qquad (22).$$

$$\lambda_s(x) = \phi_s(x), \qquad s = 1,2,\ldots\ldots\ldots\ldots, N_2, \qquad (23).$$

Obviously, the weight functions $\{\lambda\}$ and $\{\gamma\}$ must be linearly independet sets.(otherwise the equations provided by equations (18)-(21) will not be linearly independent and hence are not solvable.) Substituting equations (12) and (13) into equations (14) and (15) gives ,

$$\varepsilon_1 = \sum_{i=1}^{N_1} A\alpha_i(x)\ddot{\theta}_i(t) + \sum_{i=1}^{N_1} C\alpha_i(x)\dot{\theta}_i(t) + \sum_{i=1}^{N_1} \partial^2/\partial x^2\left[I\ddot{\alpha}_i(x)\theta_i(t)\right] -$$
$$\sum_{i=1}^{N_1} \frac{\partial}{\partial x}\left[A\left\{\sum_{k=1}^{N_2} \dot{\phi}_k(x)\psi_k(t) + \frac{1}{2}\left[\sum_{i=1}^{N_1} \dot{\alpha}_i(x)\theta_i(t)\right]^2\right\}\dot{\alpha}_i(x)\theta_i(t)\right] - m_v[g - \zeta_n\ddot{y}_3 - \zeta_n\ddot{y}_2]\delta(x - vt) \qquad (24).$$

$$\varepsilon_2 = \sum_{k=1}^{N_2} A\phi_k(x)\ddot{\psi}_k(t) - \sum_{k=1}^{N_2} (\partial/\partial x)\left[A\dot{\phi}_k(x)\psi_k(t)\right] - \frac{1}{2}(\partial/\partial x)\left\{A\left[\sum_{i=1}^{N_1} \dot{\alpha}_i(x)\theta_i(t)\right]^2\right\}, \qquad (25).$$

Substituting equations (24) and (25) into equation (18) and (20) gives

$$\sum_{i=1}^{N_1}\left[\int_0^L A\alpha_i(x)\alpha_j(x)dx\ddot{\theta}_i(t)\right]+\sum_{i=1}^{N_1}\left[\int_0^L C\alpha_i(x)\alpha_j(x)dx\dot{\theta}_i(t)\right]+\sum_{i=1}^{N_1}\left[\int_0^L(\partial^2/\partial x^2)\{I\ddot{\alpha}_i(x)\alpha_j(x)dx\theta_i(t)\}\right]$$
$$-\sum_{i=1}^{N_1}\left[\int_0^L(\partial/\partial x)\left[A\left\{\sum_{k=1}^{N_2}\dot{\phi}_k(x)\psi_k(t)+\frac{1}{2}\left[\sum_{i=1}^{N_1}\dot{\alpha}_i(x)\theta_i(t)\right]^2\right\}\dot{\alpha}_i(x)\right]\alpha_j(x)dx\theta_i(t)\right]=$$
$$\int_0^L m_v\left[g-(1-\zeta_n)\ddot{y}_3-\zeta_n(\ddot{y}_2)\right]\delta(x-vt)\alpha_j(x)dx, \qquad (26).$$

$$\sum_{k=1}^{N_2}\left[\int_0^L A\phi_k(x)\phi_s(x)dx\ddot{\psi}_k(t)\right]-\sum_{k=1}^{N_2}\left[\int_0^L(\partial/\partial x)(A\dot{\phi}_k(x)\phi_s(x)dx\psi_k(t))\right]$$
$$-\frac{1}{2}\int_0^L(\partial/\partial x)\left\{A\left[\sum_{i=1}^{N_1}\dot{\alpha}_i(x)\theta_i(t)\right]^2\right\}\phi_s(x)dx=0.0,$$
$$s=1,2,.........n1, \qquad (27)$$

Performing the integral operations (26) and (27), with account being taken of the boundary conditions, the following vector - matrix form will be obtained.

$$[MT]\{\ddot{\theta}\}+[CT]\{\dot{\theta}\}+[KT]\{\theta\}=\{F\}, \qquad (28)$$

$$[MA]\{\ddot{\psi}\}+[KA]\{\psi\}=\{PF\}, \qquad (29)$$

Where [MA] is the axial mass matrix, [KA] is the axial stiffness matrix, {PF} is the axial force vector which is generated by the transverse deflections, [MT] is the transverse mass matrix, [KT] is the stiffness matrix and {F} is the load vector generated by the moving load. The elements of their vectors and matrices are respectively given by:

$$[MA]=\int_0^1[\phi_k(x)A\phi_s(x)]dx, \qquad k,s=1,2,..............,N_2,$$
$$[KA]=\int_0^1[\dot{\phi}_K(x)A\dot{\phi}_s(x)]dx,$$
$$\{PF\}=-\frac{1}{2}\int_0^1\left[\dot{\phi}_K(x)A\{\theta\}^T\dot{\alpha}_i(x)\dot{\alpha}_j(x)\{\theta\}\right]dx,$$

$$[MT]=\int_0^l \alpha_i(x)A\alpha_j(x)dx, \qquad i,j=1,2,\ldots\ldots\ldots\ldots,N_1,$$

$$[KT]=[KL]+[KG]$$

$$[KL]=\int_0^l \ddot{\alpha}_i(x)I\ddot{\alpha}_j(x)dx,$$

$$[KG]=\int_0^L\left[\dot{\alpha}_i(x)\left\{A\left[\dot{\alpha}_j(x)\{\psi\}+\frac{1}{2}\{\theta\}^T\dot{\alpha}_i(x)\dot{\alpha}_j(x)\{\theta\}\right]\right\}\dot{\alpha}_j(x)\right]dx,$$

$$[CT]=a[MT]+b[KT], \qquad \text{a,b are constants}$$

$$\{F\}=\int_0^l\{m_v[g-(1-\zeta_n)\ddot{y}_3-\zeta_n(\ddot{y}_2)]\}\delta(x-vt)\alpha_j(x)dx,$$

where [KL] is the linear stiffness matrix , superscript T denotes matrix transposition, [KG] is the geometric stiffness matrix. Substituting equation (12) into equation (10) gives the equation of motion for the moving load as :

$$\zeta_n\ddot{y}(t)+(1-\zeta_n)\ddot{y}_3(t)+\lambda^2y_2(t)-\lambda^2\left[\sum_{i=1}^{N_1}\alpha_i(vt)\theta_i(t)\right]=0.0, \qquad \text{at} \quad x=vt, \qquad (30)$$

By combining equations (11), (28) and (30),we will have the following coupled equation (The word "coupled" is used to imply that the same dependent variables appear in more than one equation of the set, and therefore no equation can be solved independent of the other in the set) :

$$\begin{bmatrix}[MT] & m_v\zeta_n\alpha_i(vt) & m_v(1-\zeta_n)\alpha_i(vt)\\ 0 & \zeta_n & (1-\zeta_n)\\ 0 & 0 & (1-\zeta_n)\end{bmatrix}\begin{Bmatrix}\ddot{\theta}_i\\ \ddot{y}_2\\ \ddot{y}_3\end{Bmatrix}+\begin{bmatrix}[CT] & 0 & 0\\ 0 & 0 & 0\\ 0 & -2\hbar\xi & 2\hbar\xi\end{bmatrix}\begin{Bmatrix}\dot{\theta}_i\\ \dot{y}_2\\ \dot{y}_3\end{Bmatrix}$$

$$+\begin{bmatrix}[KT] & 0 & 0\\ -\lambda^2\alpha_j(vt) & \lambda^2 & 0\\ 0 & -\zeta^2 & -\zeta^2\end{bmatrix}\begin{Bmatrix}\theta_i\\ y_2\\ y_3\end{Bmatrix}=\begin{Bmatrix}m_vg\alpha_i(vt)\\ 0\\ 0\end{Bmatrix} \qquad (31)$$

which is in the standard form of the equation of motion of dynamical systems

$$M\ddot{x}+C\dot{x}+Kx=f(t), \qquad (32)$$

The initial conditions $\theta(0), \dot{\theta}(0), \psi(0)$ and $\dot{\psi}(0)$, are respectively, determined by substituting the approximate solutions (12) and (13) into residual equations (16) and (17), and then the obtained residuals are substituted into the integral equations (19) and (21). Which leads to the following ,equations

$$\int_0^1 \left[\sum_{i=1}^{N_1} \left(\alpha_i(x)\theta_i(0) - w_0(x) \right) \right] \alpha_j(x)dx = 0.0, \qquad j = 1,2,........,N_1,$$

$$\int_0^1 \left[\sum_{i=1}^{N_1} \left(\alpha_i(x)\dot{\theta}_i(0) \right) - \dot{w}_0(x) \right] \alpha_j(x)dx = 0.0,$$

$$\int_0^1 \left[\sum_{k=1}^{N_2} \phi_K(x)\psi_k(0) - u_0(x) \right] \phi_s(x)dx = 0.0, \qquad s = 1,2,3,..........,N_2,$$

$$\int_0^L \left[\sum_{k=1}^{N_2} \phi_k(x)\dot{\psi}_k(0) - \dot{u}_0(x) \right] \phi_s(x)dx = 0.0,$$

Then

$$\theta(0) = P^{-1}d, \qquad \dot{\theta}(0) = P^{-1}\dot{d}, \qquad \psi(0) = Z^{-1}b, \qquad \dot{\psi}(0) = Z^{-1}\dot{b},$$

Where d and b are column vectors, P and Z are $N_1 * N_1$ & $N_2 * N_2$ matrices, respectively, their elements are :

$$P_{ji} = \int_0^1 \alpha_i(x)\alpha_j(x)dx, \qquad d_j = \int_0^1 w_0(x)\alpha_j(x)dx, \qquad \dot{d}_j = \int_0^1 \dot{w}_0(x)\alpha_j(x)dx,$$

$$Z_{ks} = \int_0^1 \phi_s(x)\phi_k(x)dx, \qquad b_k = \int_0^1 u_0(x)\phi_k(x)dx, \qquad \dot{b}_k = \int_0^1 \dot{u}_0(x)\phi_k(x)dx,$$

$$i,j = 1,2,3,...........,N_1, \qquad s,k = 1,2,3,...............,N_2,$$

CHAPTER THREE

METHOD OF SOLUTION

3.1 LINEARIZING APPROXIMATION

It is generally suitable for multi-dimensional systems of non-linear equations to be linearized by incremental displacement [27]. Equation (31), at time (t+dt) is expressed as :

$$[M]\{\ddot{\Re}\}^{t+\Delta t}+[C]\{\dot{\Re}\}^{t+\Delta t}+\left\{R\left(\{\Re\}^{t+\Delta t}\right)\right\}=\{F\}^{t+\Delta t}, \qquad (33)$$

Where $\{\Re\}=\{\theta, y_2, y_3\}^T$, and $\{R\}$ is a vector of restoring forces that depends upon the displacement field. If the vector $\{R\}$ is differentiable in the neighbourhood of all deformed shapes $\{R\}$, then the expansion

$$\left\{R\left(\{\Re\}^{t+\Delta t}\right)\right\}=\left\{R\left(\{\Re\}^{t}\right)\right\}+\left(\partial\{R\}/\partial\{\Re\}\right)\downarrow_{\{\Re\}=\{\Re\}^{t}}\{\Delta\Re\}$$
$$+\frac{1}{2}\left(\partial^2\{R\}/\partial\{\Re\}^2\right)\downarrow_{\{\Re\}=\{\Re\}^{t}}\{\Delta\Re\}^2+\ldots\ldots \qquad (34).$$

is obtained where $\{\Delta\Re\}=\{\Re\}^{+\Delta}-\{\Re\}$ is the incremental displacement. Substituting equation (34) into equation (33) , and by defining the tangent stiffness matrix as :

$$[K]^t=\partial\{R\}/\partial\{\Re\}\downarrow_{\{\Re\}=\{\Re\}^{t}} \qquad (35)$$

and neglecting the higher-order terms beyond the secound derivatives, gives the linearized equation.

$$[M]\{\ddot{\Re}\}^{t+\Delta t}+[C]\{\dot{\Re}\}^{t+\Delta t}+[K]^t\{\Delta\Re\}=\{F\}^{t+\Delta t}-\left\{R\left(\{\Re\}^{t}\right)\right\}, \qquad (36)$$

3.2 NEWMARK METHOD

In time-dependent (unsteady) problems, the undetermined parameters $\theta_i(t)$ and $\psi_k(t)$ in equations (12) and (13) are assumed to be function of time, while $\alpha_i(x)$ and $\phi_k(x)$ are assumed to depend on spatial coordinates. This leads to two stages of solution, both of which employ approximate methods. In the solution of time-dependent problems, the spatial approximation is considered first and the time (or time like) approximation next. Such a procedure is commonly known as semidiscrete approximation (in space). Semidiscrete variational approximation in space results in a set of ordinary differential equations in time, which must be further approximated to obtain a set of algebric equations. The spatial approximation of time dependent problems leads to matrix differential equations (in time) of the form as equations (28),(29) and (36) (Linearized equation).

In structural dynamics problems the equations of motion involve the second-order time derivatives of the dependent variables .The semidiscrete (spatial) approximation of the equations results in matrix differential equation of the form of equation (32). There are several approximation schemes available for time derivative. The most commonly used one is the Newmark direct integration method . In the Newmark method the first time derivative $\{\dot{\Re}\}$ and the function (of time) $\{\Re\}$ itself are approximated at (t+dt)th time step by the following expressions :

$$\{\dot{\Re}\}^{t+\Delta t} = \{\dot{\Re}\}^{t} + \left[(1-\alpha)\{\ddot{\Re}\}^{t} + \alpha\{\ddot{\Re}\}^{t+\Delta t}\right]\Delta t, \qquad (37)$$

$$\{\Re\}^{t+\Delta t} = \{\Re\}^{t} + \{\dot{\Re}\}^{t}\Delta t + \left[(1/2-\beta)\{\ddot{\Re}\}^{t} + \beta\{\ddot{\Re}\}^{t+\Delta t}\right](\Delta t)^2, \qquad (38).$$

where α and β are parameters that control the accuracy and stability of the scheme. The choice $\alpha = 1/2$ and $\beta = 1/4$ is known to give an unconditionally stable scheme,which corresponds to the constant-average-acceleration method.

By writing equation (38) for $\{\ddot{\Re}\}^{t+\Delta t}$ in terms of $\{\Re\}^{t+\Delta t}$, then substituting $\{\ddot{\Re}\}^{t+\Delta t}$ into equation (37) to obtain equations for $\{\dot{\Re}\}^{t+dt}$ and $\{\ddot{\Re}\}^{t+dt}$, each in terms of $\{\Re\}^{t+\Delta t}$ only. Finally, substituting the expressions thus obtained into equation (36) and collecting coefficients of $\{\Delta\Re\}$ and known quantities on the right-hand side of the equality sign, we will obtain:

$$[\hat{K}]\{\Delta\Re\} = \{\hat{F}\}^{t+\Delta t}, \quad (39).$$

where

$$[\hat{K}] = [K] + A_0[M] + A_1[C],$$

$$\{\hat{F}\}^{t+\Delta t} = \{F\}^{t+dt} + [M]\left[A_2\{\dot{\Re}\}^t + A_3\{\Re\}^t\right] + [C]\left[A_4\{\dot{\Re}\}^t + A_5\{\ddot{\Re}\}^t\right] - \{R\}^t,$$

where

$A_0 = 1/\beta(\Delta t)^2$, $\quad A_1 = \alpha/\beta(\Delta t)$, $\quad A_2 = 1/\beta(\Delta t)$, $\quad A_3 = 1/(2\beta) - 1$

$A_4 = (\alpha/\beta) - 1$, $\quad A_5 = \Delta t[\alpha/\beta - 2]/2$,

Once the solution $\{\Delta\Re\}$ is known at (t+dt) the first and second derivatives (velocity and acceleration) of $\{\Re\}$ at (t+dt) can be computed from the following equations :

$$\{\ddot{\Re}\}^{t+\Delta t} = A_0\left(\{\Re\}^{t+\Delta t} - \{\Re\}^t\right) - A_2\{\dot{\Re}\} - A_3\{\ddot{\Re}\}, \quad (40)$$

$$\{\dot{\Re}\}^{t+\Delta t} = \{\dot{\Re}\}^t + A_9\{\ddot{\Re}\}^t + A_{10}\{\ddot{\Re}\}^{t+\Delta t}, \quad (41)$$

where

$$A_9 = (1-\alpha)\Delta t, \qquad A_{10} = \alpha(\Delta t),$$

Equations (40) and (41) are obtained from rearranging the expressions in equations (37) and (38). For a given set of initial conditions $\{\Re\}_0, \{\dot{\Re}\}_0$ and $\{\ddot{\Re}\}_0$ we can solve equation (39) repeatedly , marching forward in time , for the column vector $\{\Re\}$ and its time derivatives at any time $t \rangle 0$.

It must be pointed out that one can expect better results if smaller time steps are used. In practice , however , one wishes to take as large a time step as possible to cut down the computational expense. Larger time steps,in addition to decreasing the accuracy of the solution, can introduce some unwanted ,numerically induced oscillations into the solution. Thus an estimate of an upper bound on the time step proves to be very useful.

A couple of comments are stated on the selection of the time step and the computation of the initial conditions . Although the Newmark method is unconditionally stable (i.e., the solution is stable for any value of dt ; however, it may be inaccurate), it is helpful to have a means to determine the value of dt for which the solution is also accurate. The following formula gives an estimate for the time increment :

$$\Delta t = \frac{T_{min}}{\pi}$$

where T_{min} smallest period of natural vibration associated with the approximate problem. An estimate for Δt can also be obtained from condition that the smallest eigenvalue of the eigenvalue problem

$$\left(A_0[M]-\lambda[\hat{K}]\right)\{W\}=0.0$$

is less than 1.

where λ is minimum eigenvalue and it is equal to

$0<\lambda<1$ stable without oscillations

$-1<\lambda<0$ stable with oscillations

$\lambda<1$ unstable

3.3 TRANSITIONAL ANALYSIS

The transitional responses of the derived system are calculated by using the Newmark method . Before the incremental solution is carried out, the linear constant structure matrices (i.e., the linearized effective stiffness, linear stiffness, mass and damping matrices) and the load vectors are assembled . During the step-by-step solution, the linearized effective stiffness matrix is updated for the non-linearity in the system.

The incremental equilibrium equations at time t+dt are

The incremental equilibrium equations at time t+dt are

$$[M]\{\ddot{\Re}\}^{t+\Delta t}[C]\{\dot{\Re}\}^{t+\Delta t}+[K]^{t}\{\partial\Re\}=\{F\}^{t+\Delta t}-\{R\}^{t}, \tag{42}$$

$$[MA]\{\ddot{\psi}\}^{t+\Delta t}+[KA]\{d\psi\}=\{PF\}^{t+\Delta t}-[KA]\{\psi\}^{t}, \tag{43}$$

To improve the solution accuracy of the non-linear equation (42), it is necessary to carry out the equilibrium iteration in each time step. The equilibrium equation is obtained as

$$[M]\{\ddot{\Re}\}_{i}^{t+\Delta t}+[C]\{\dot{\Re}\}_{i}^{t+\Delta t}+[K]_{(i-1)}^{t}\{\delta\Re\}_{i}=\{F\}^{t+\Delta t}-\{R\}_{(i-1)}^{t+\Delta t}, \qquad i=1,2,3,\ldots \ldots, \tag{44}$$

where

$$\{\ddot{\Re}\}_{i}^{t+\Delta t},\{\dot{\Re}\}_{i}^{t+\Delta t}\ \&\ \{\Re\}_{i}^{t+\Delta t}=\{\Re\}_{i-1}^{t+\Delta t}+\{\delta\Re\}_{i},$$

are vectors of the accelerations, velocities and deflections at the time i^{th} interation, respectively. The iterative compution is continued untill

$$\left\|\{\delta\Re\}_{i}\right\|/\left\|\{\Re\}_{i}^{t+dt}\right\|\leq \text{tol}.$$

is satisfied where *tol.* denotes the tolerances and $\|.\|$ denotes the Euclidean norm. The transitions of the accelerations, velocities and deflections from t to t+dt are given in Appendix II. Once the transverse deflection , $\{\Re\}^{t+dt}$, is known at time t+dt . $\{PF\}^{t+dt}$ can be calculated. As $\{PF\}^{t+dt}$ is regarded as the axial force vector which is generated by transverse deflections, the longitudinal deflections, velocities and accelerations, which correspond to $\{\psi\}^{t+dt},\{\dot{\psi}\}^{t+dt}$ and $\{\ddot{\psi}\}^{t+dt}$ respectively, at time t+dt are obtained.

CHAPTER FOUR

NUMERICAL EXAMPLE AND DISCSSION

The classical variational methods(i.e.,Ritz, Galerkin , Least-squares, etc.) provide simple means of finding approximate solutions to physical problems. The formulative and computational efforts involved are less compared to most other methods ,such as the finite-difference and the finite-element methods. Furthermore,the approximate solutions obtained are continuous functions of position in the domain. The main disadvantage, from the practical point of view, of the variational methods that prevented them from being cometitive with other methods is the difficulty encountered in selecting the approximation functions. A part from the properties the functions are required to satisfy ,there exists no systematic procedure of constructing them. The selection process becomes more difficult when the domain is geometrically complex and/or the boundary conditions are complicated. If the functions are not selected from the domain space of the operator of the equation being solved, the resulting solution could be either zero or wrong. One cannot automtize the procedure for a given equation because the choice of approximation functions differs with the boundary conditions.

4.1 NUMERICAL EXAMPLE

For use in the numerical calculations, the approximating shape functions used in equations (12) and (13) are ,respectively, given by [12,16,24]

$$\alpha_i(x) = \sin(i\pi x/l),), \quad i = 1,2,\ldots\ldots,N_1 \qquad (45a),$$
$$\phi_k(x) = \sin(k\pi x/l), \quad K = 1,2,\ldots\ldots,N_2 \qquad (45b),$$

The initial conditions for the transverse and longitudinal deflections for the beam and for the vertical deflections of the moving load are assumed to be :

$$w_0(x) = \dot{w}_0(x) = 0.0 \qquad u_0(x) = \dot{u}_0(x) = 0.0,$$
$$y_{2_0} = \dot{y}_{2_0} = 0.0 \qquad y_{3_0} = \dot{y}_{3_0} = 0.0 \qquad (46).$$

The dimensionless variables to be used for the beam and for the moving load are taken as [23].

$g = .49 * 10^{-5}$ $\xi = ..005$ $\lambda = .5$

$\zeta_n = .02$ $\hbar = .25\sqrt{1-\zeta_n}$

and the values of α and β , which are parameters of the Newmark method, are determined as .5 and .25 , respectively, by considering numerical accuracy and stability.

The geometry of the simply supported beam which is used is shown in Figure 2. where the cross-sectional area, A(x) , and the moment of inertia , I(x), are assumed to be [9,23,26] :

$$A(x) = A_m(-\beta_1|x-.5|+1.), \quad (47)$$
$$I(x) = I_m(-\beta_1|x-.5|+1.),$$

Where β_1 is constant $(0 \le \beta_1 \le 1)$,and A_m and I_m are positive constants. The values of A, I and $r = \sqrt{I/A}$ (radius of gyration)are denoted by A_m, I_m and r_m at mid-span . The load parameter q_s [23] is defined by

$$q_s = \frac{m_v g}{48 I_m} \quad (48)$$

which relates to the linear static deflection

Numerical convergence experiments were performed with various time steps, dt . When the time step was taken to be less than 2.0, the numerical accuracy of the deflections , velocities and accelerations improved only slightly , but the computational load was considerably increased . Therefore, in the subsequent calculations, the value of dt was set to 2.0 and the number of terms used in the approximations of the solutions N_1 and N_2 was set to three .

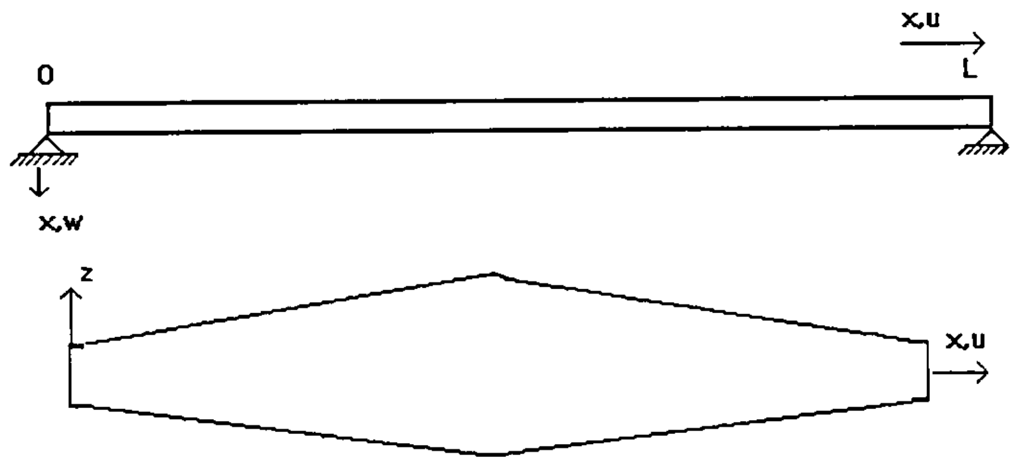

Figure 2. Geometry of beam [9,23,28].

4.2 DISCSSION

Approximate solutions for equations (28) and (29) were determined by using Newmark method. The transient responses at mid-span for the simply supported beam are shown in Figures 3(1)-(14), where deflection ratio is plotted against position ratio. In Figures 3(1-7) the non-linear responses is compared with the linear responses (the linear response values were obtained by neglecting the first term on the right-hand of equation (1)). It is seen from these Figures that the amplitudes of the non-linear deflections are somewhat smaller than these for the linear deflections, and this is due to the fact that the inplane forces increase the bending stiffness of the beam and therefore it reduces the bending deflections. After the vehicle finish the transit of the beam, where the transit times equal to v/L, the beams goes into damped vibration.

Figures 3(1-3) shows the transverse deflection for non-uniform beam (β_t =1.0), radius of gyration at mid-span , r_m , is .01, v = .002 , C=0.0 , and q_s=.005,.0055 and .006, respectively. To easy the comparison, Figure 3(4) shows the difference between the non-linear deflections as the load parameter, q_s , is increased . It is seen from this Figure (3(4)) that the amplitude of the deflections becomes larger if the load parameter is larger . This has been observed by many investigators such as Hino *et al* [22-24]. In Figures 3(5-7) , the transverse deflections at mid-span are shown , the velocities being given by .002 , .003 and .004, respectively , β_l = 0.0 , r_m=.01 , C=0.0 and the load parameter by .005. To easy the comparison, Figure 3(8) shows the relations between the non-linear transverse deflection and T (Time) for different velocities. It is seen from this Figure that the amplitude becomes larger if the moving load velocity is faster . This has been observed by Hino *et al* [22] and Hino *et al* [23] . Figure 3(9) shows the non-linear deflections for uniform (β_l = 0.0) and non-uniform beams (β_l= 1.0). It is seen from this Figure that when uniform beams are considered , the effects of longitudinal deflections and inertia are to reduce the non-linearity . This has been observed by Raju *et al* [9] , Mei [29] and Hino et al [23].

1- The transient responses at mid-span for the simply supported beam which is shown in Figure 3(10) was constructed for the sake of comparison with previouslly published results (which was obtained by using different method [23]). It was found out that the obtained results by using the Galerkin method are almost consistent with results of [23]. As stated earlier the changes in response due to variations in the damping coefficient of the beam material will be investigated , Figures 3(11-12) were constructed for showing the above . To easy the comparison, Figure 3(13)-(14) shows the non-linear deflection behaviour for different values of C (damping coefficient of the beam material). It is seen from this Figure that by increasing the value of C the free damped vibration, which is obtained after the vehicle cross the beam , is reduced more than the forced damping.

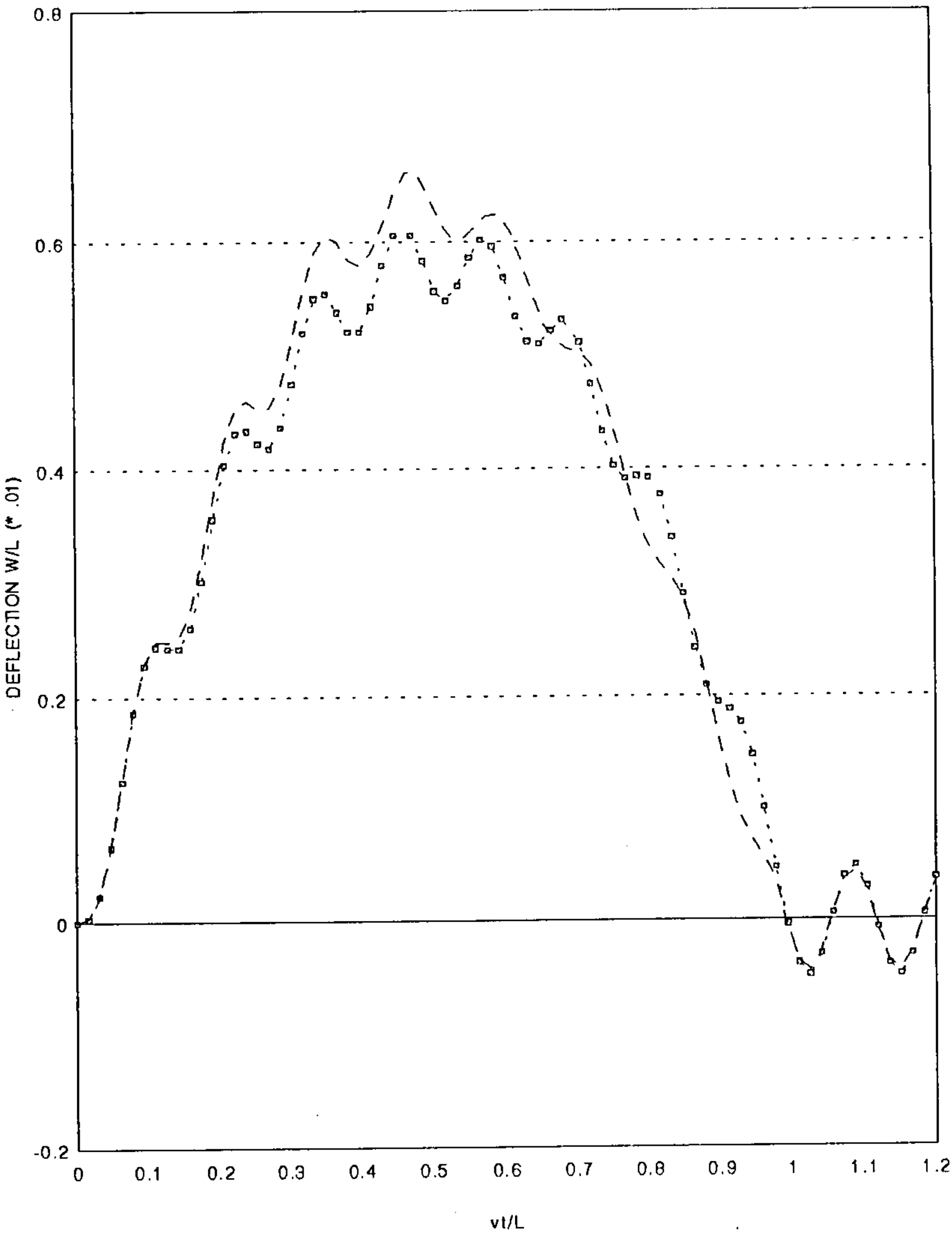

Figure 3(2). Transverse deflection at mid-span for simply supported beam
ß1=1.0, Rm=.01 , Qs=.0055, C=0.0, v=.002,t=time.
- - - - - -;linear model, ...;non-linear model.

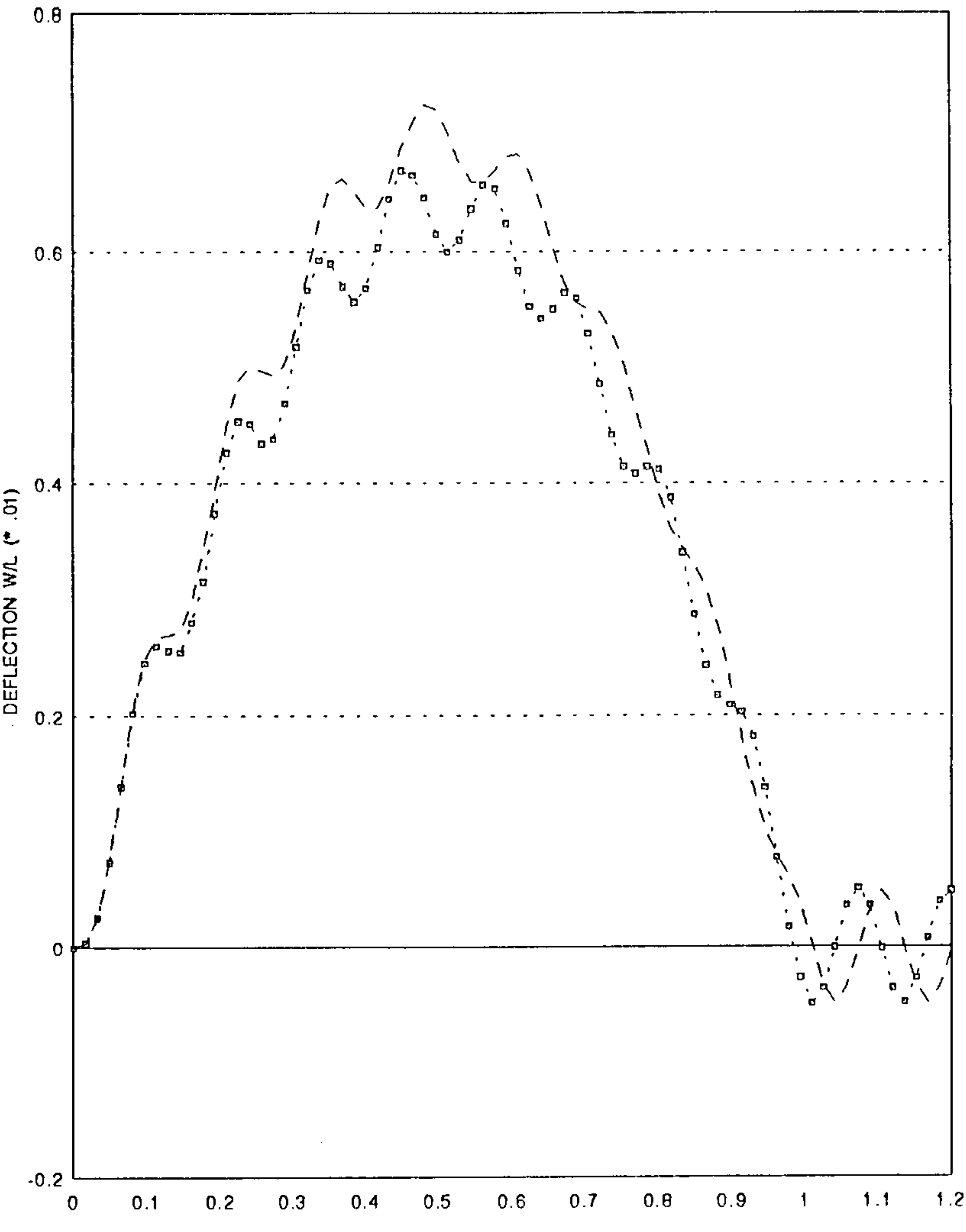

Figure 3(3). Transverse deflection at mid-span for simply supported beam . β1=1.0 , Rm=.01, Qs=.006, v=.002, C=0.0.
- - - - -;linear model,. . . . . .;non-linear model

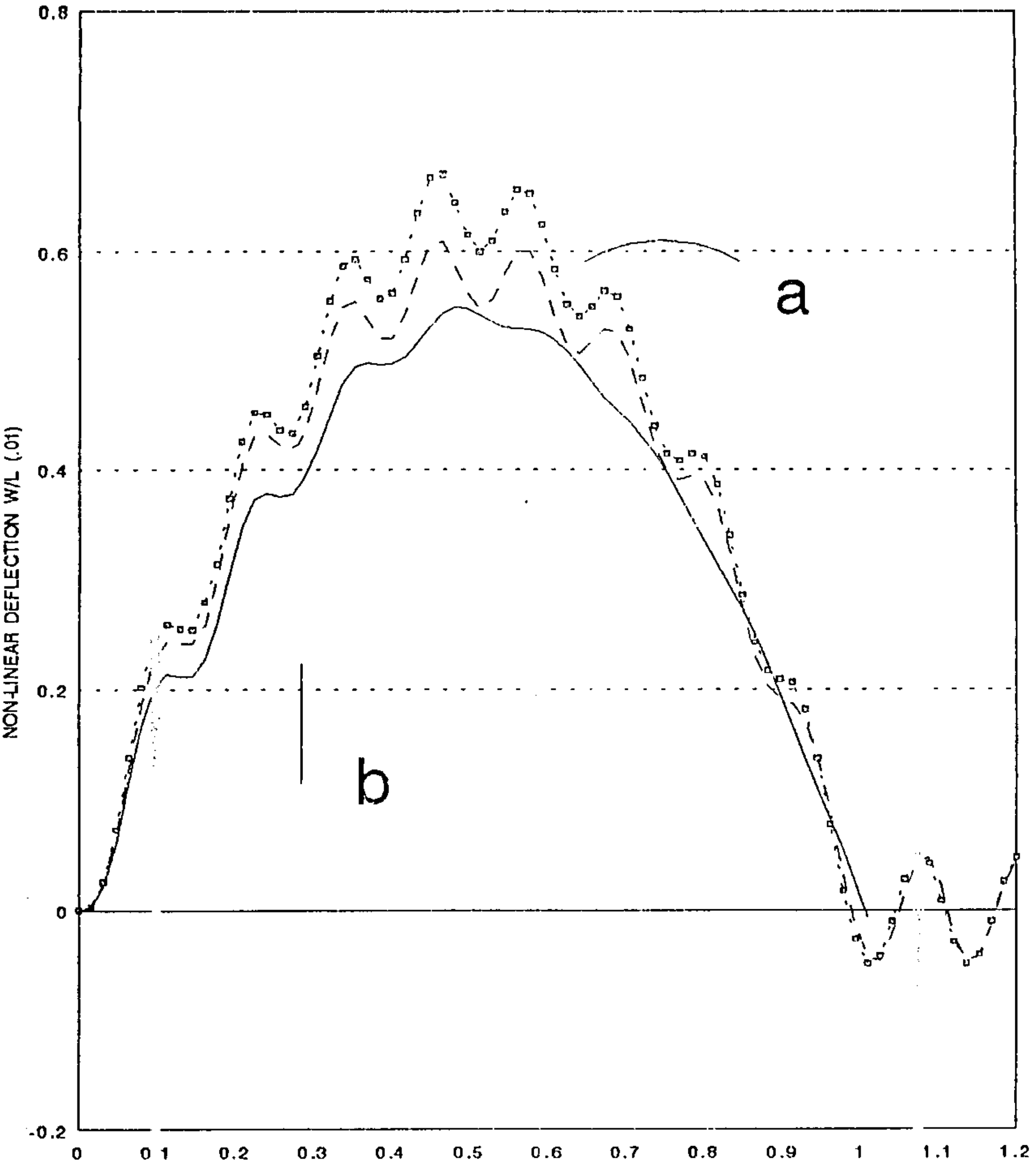

Figure 3(4). Non-linear deflection at mid-span for simply supported beam B1=1.0,Rm=.01,V=.002,C=0.0. (a) Qs=.006; (b) Qs=.0055; (c) Qs=.005.

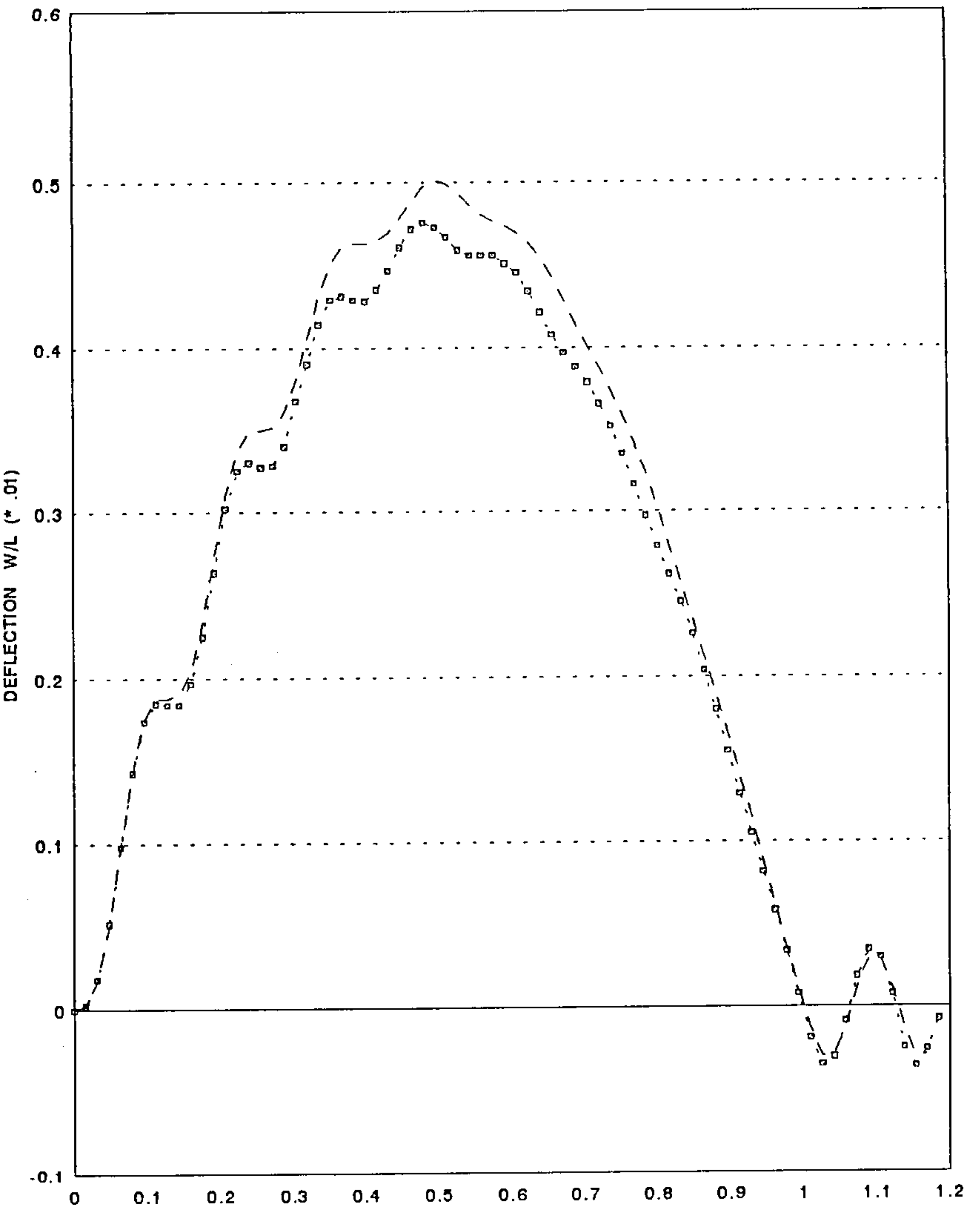

Figure 3(5). Transverse deflection at mid-span for simply supported beam. β1=0.0, Rm=.01, Qs=.005, v=.002, C=0.0
- - - -;linear model, ...; non-linear model.

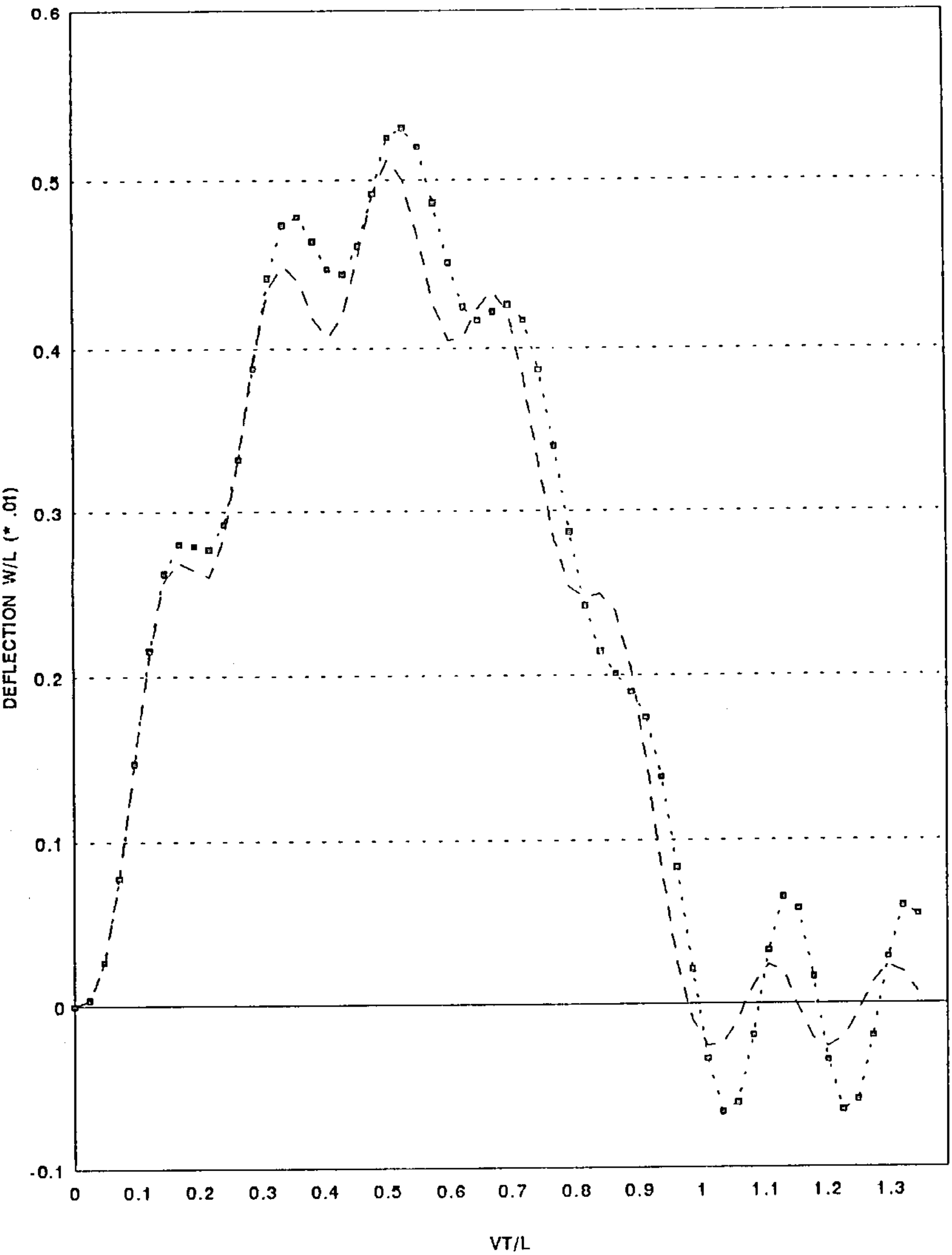

Figure 3(6). Transverse deflection at mid-span for simply supported beam. θ_1=0.0 , Rm=.01, Qs=.005, C=0.0, v=.003 . . . . . . ;linear model, - - - - ; non-linear model.

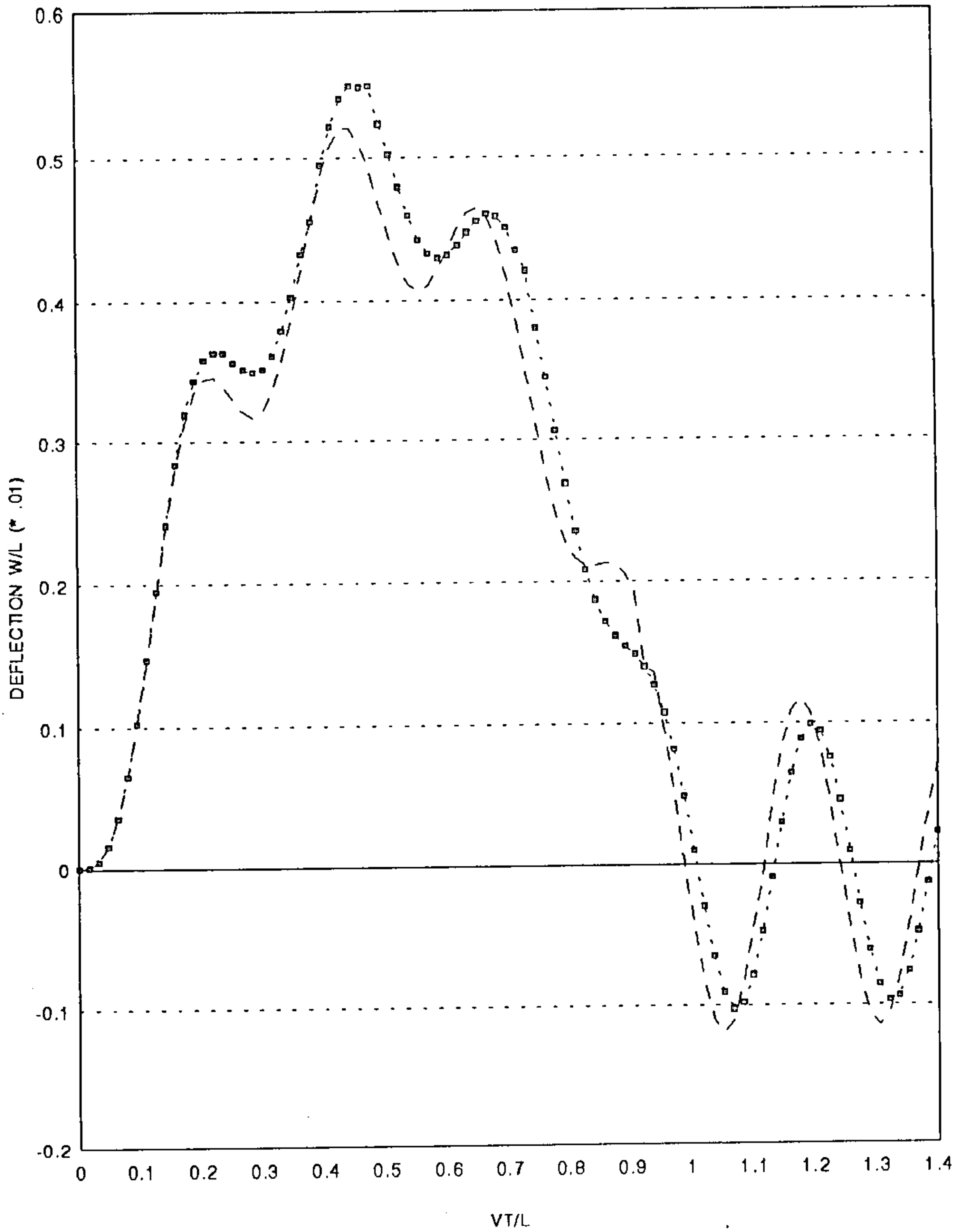

Figure 3(7). Transverse deflection at mid-span for simply supported beam. β1=0.0, Rm=.01, Qs=.005, C=0.0, V=.004,
. . . . ; linear model, - - - - -; non-linear mpdel.

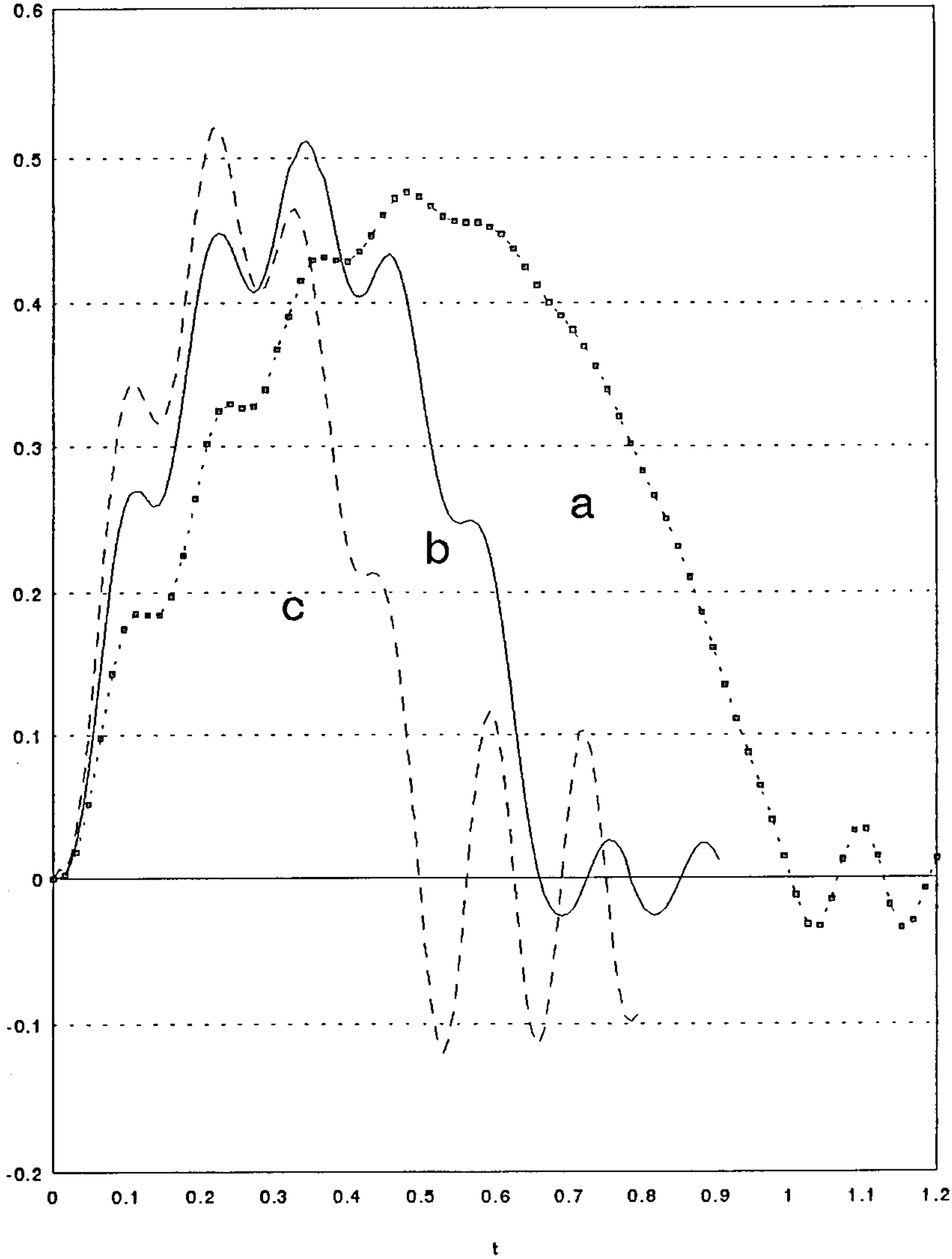

Figure 3(8). Non-linear transverse deflection at mid-span for simply supported beam. ß1=0.0, Rm=.01, Qs= .005,C=0.0 (a); v=.002, (b); v=.003, (c) v=.004.

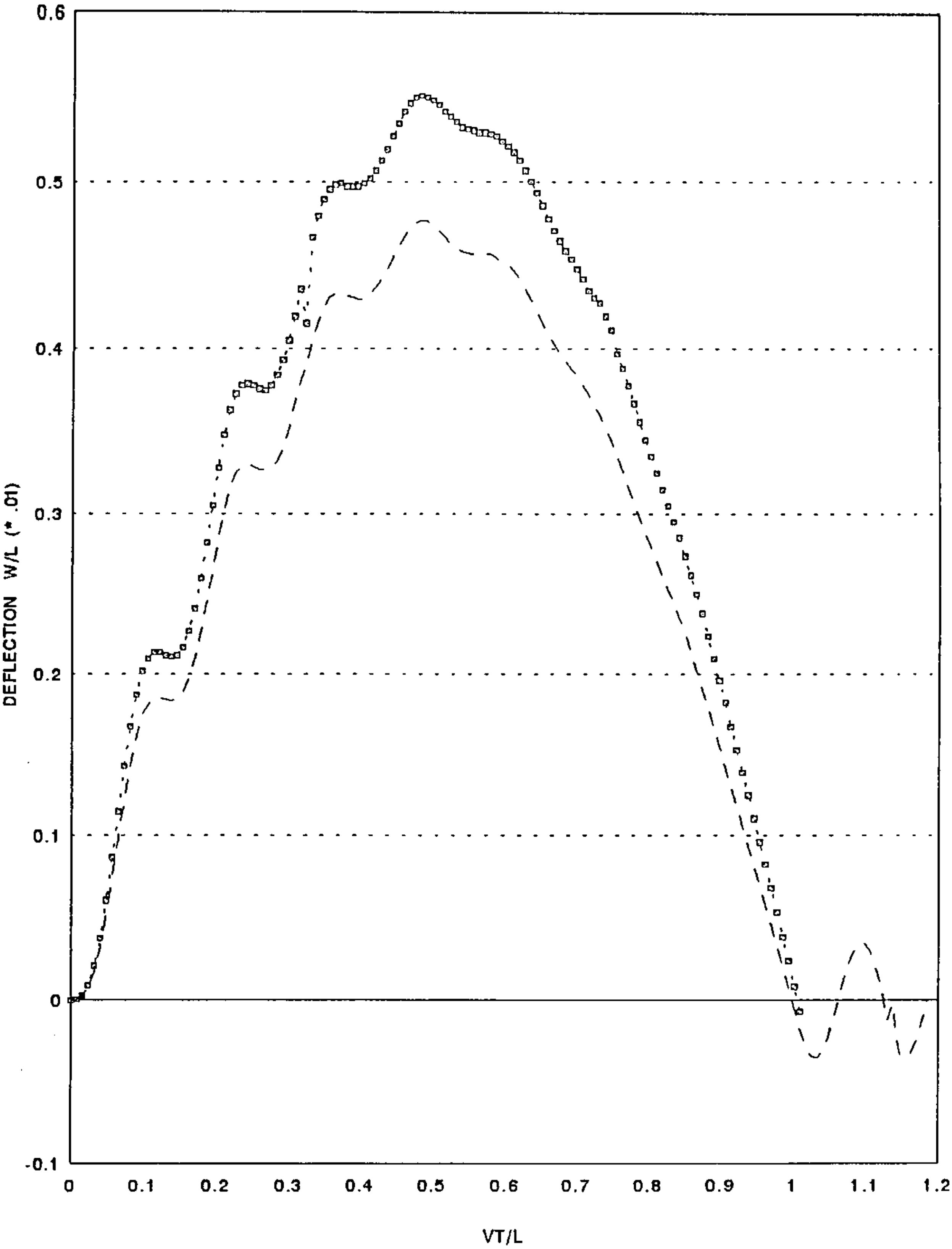

Figure 3(9). Transverse deflection at mid-span for simply supported beam. Qs=.005, v=.002, C=0.0, Rm=.01, ...;β1=1.0, - - - - -;β1=0.0.

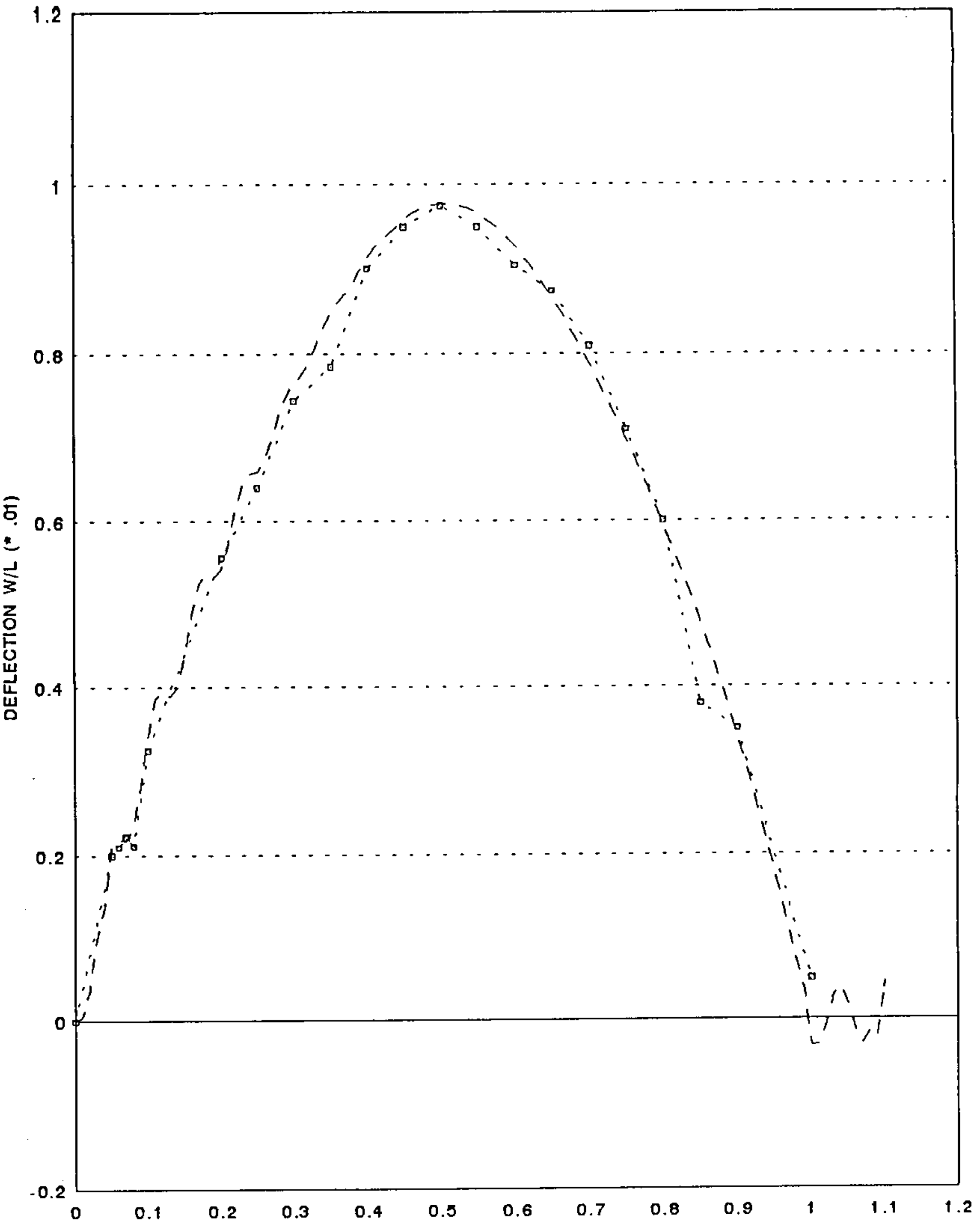

Figure 3(10). Transverse deflection at mid-span for simply supported beam. B1= 1.0, Rm=.01, Qs=.01, v=.001, C=0.0
. . . .,result [23]; - - - - -;present

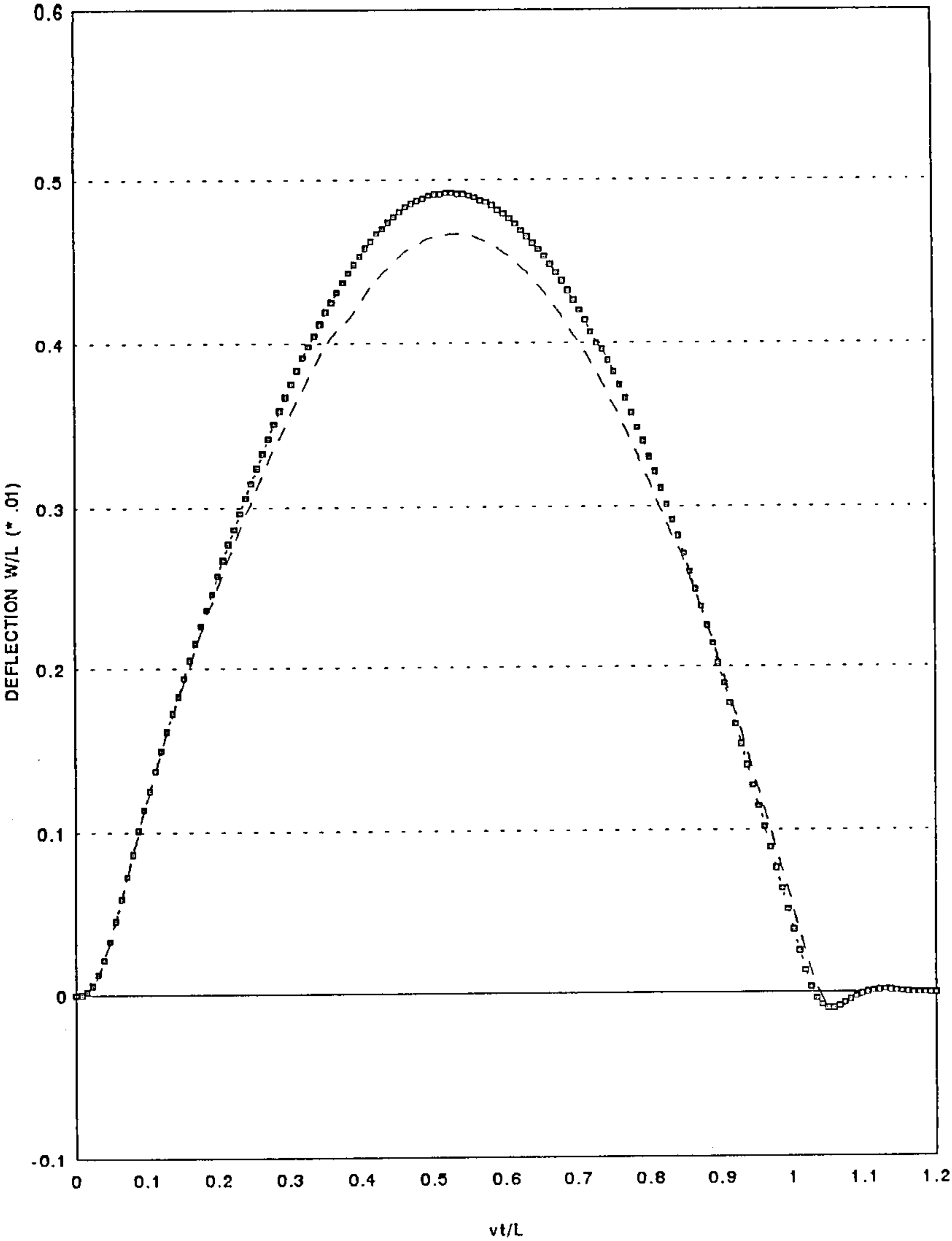

Figure 3(11). Transverse deflection at mid-span for simply supported beam. β1=0.0, Rm=.01, Qs=.005 , v=.002, C =.1.
. . . . . .;linear , - - - - -;non-linear model.

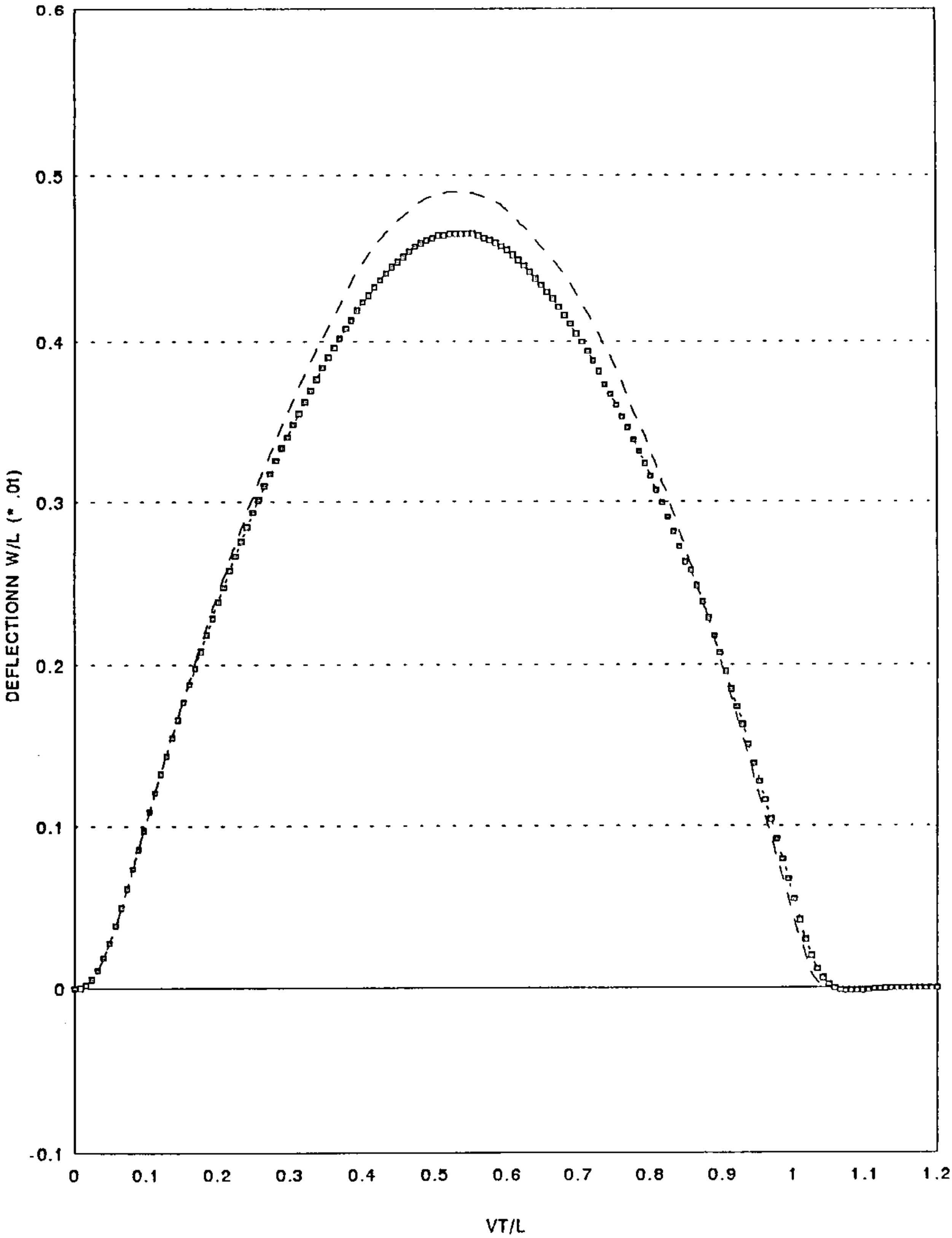

Figure 3(12). Transverse deflection at mid-span for simply supported beam. ß1=0.0, Rm=.01, Qs=.005, v=.002, C=.15.
- - - -,linear; . . . . . .,non-linear model.

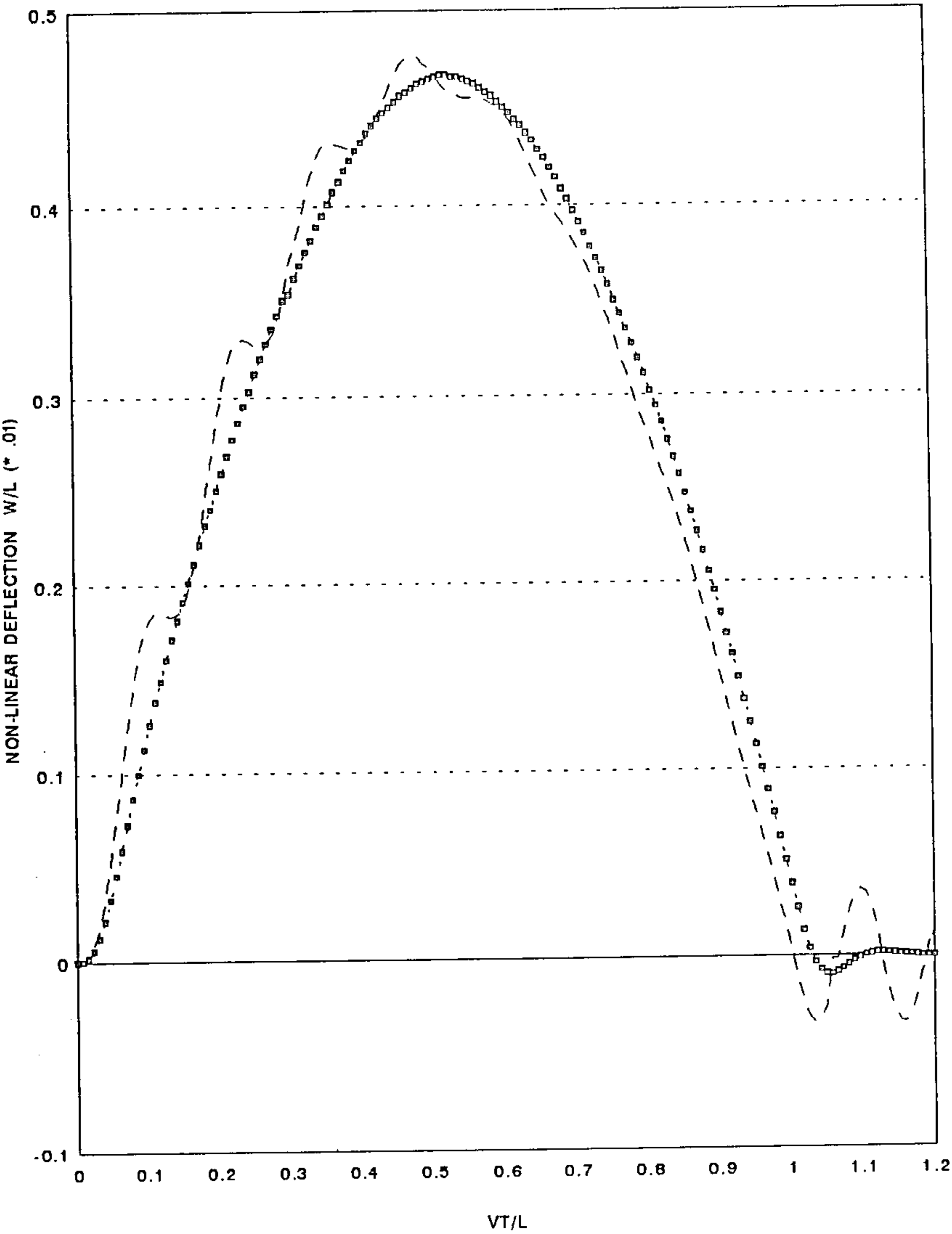

Figure 3(13). Non-linear deflection at mid-span for simply supported beam. ß1=0.0, Rm=.01, Qs=.005, v=.002
. . . .; C=.1 , - - - - - - ;C=0.0.

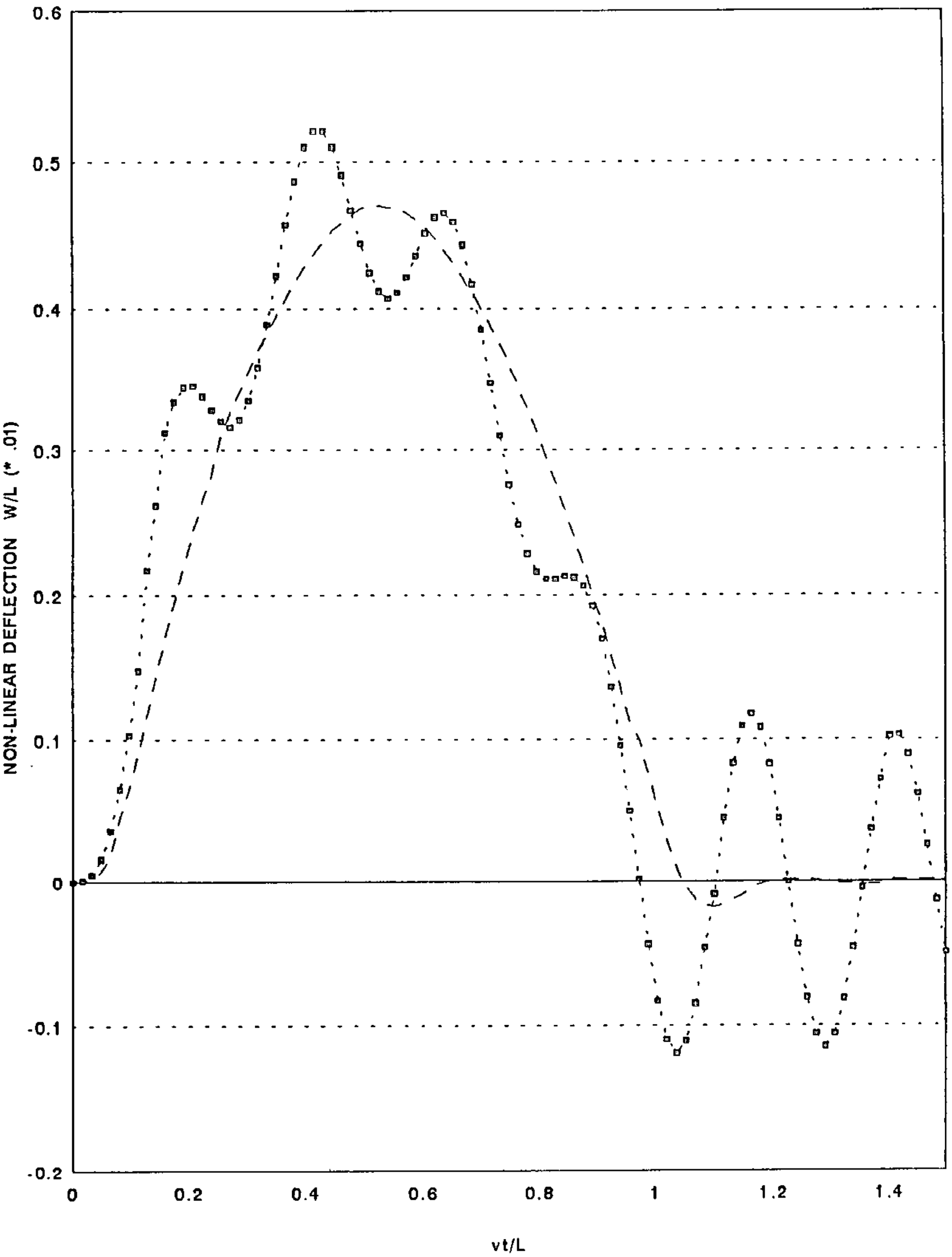

Figure 3(14). Non-linear deflection at mid-span for simply supported beam. ß1=0.0, Rm=.01, Qs=.005, v=.004, ..., C=0.0; - - - - - - -,C=.1 .

CONCLUSION

In this paper , the dynamic deflection of a beam and moving vehicle have been analysed by using the variational method.The formulation in the spatial domain has been carried out by using the Galerkin form of the method of weighted residuals (MWR). The non-linear equations derived were then linearized by using the incremental method ,and the transient responses computed by the Newmark method . The vehicle making up the moving load on the beam have been assumed to be a two-degree of freedom. The responses at mid-span for the non-linear model have the smallest amplitude, as compared with responses for the linear model. Only a few terms of the Galerkin series solution were needed for convergence in the calculations. Thus the MWR is a useful practical method of calculation for this problem (if the correct approximation function is used).

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APPENDIX I

EQUATION OF MOTION

In this Appendix, the governing differential equations of a beam and the moving vehical are derived by suitable manipulation of the differetial equations and boundary conditions arising from a variation of the total potential.

1. For the beam motion :

The non-linear strain displacement relations of a beam are(see reference [6])

$$\varepsilon_x = \frac{1}{2}(\partial w/\partial x)^2 + \partial u/\partial x \qquad \text{A1}$$

$$\psi_X = \partial^2 w/\partial x^2 \qquad \text{A2}$$

where ε_x is the axial strain , ψ_x is the curvature, and u and w are the axial and transverse displacements, respectively. Using Hooke's law we can write the following relation for the elastic strain energy U:

$$U = \int_0^L \left(\frac{1}{2} EA\varepsilon^2{}_x + \frac{1}{2} EI\psi^2{}_x + S\varepsilon_x \right) dx$$

$$= \int_0^L \left\{ EA\left[\partial u/\partial x + \frac{1}{2} \; / \; \right] \right\}$$

$$= \int_0^L \left\{ EA/2\left[\partial u/\partial x + 1/2(\partial w/\partial x)^2\right] + EI/2(\partial^2 w/\partial x^2)^2 + S\left[\partial u/\partial x + \frac{1}{2}(\partial w/\partial x)^2\right]\right\} dx$$

A3

Here E,A,I,S and L are Young's modulus , the area of the cross-section , the moment of inertia of the cross-section, the initial axial force, and the length of the beam,respectively.

The kinetic energy of the beam is given by :

$$T = \frac{m}{2}\int_0^L \left[(\partial u/\partial t)^2 + (\partial w/\partial t)^2\right] dx \qquad \text{A4}$$

where m is the mass per unit length and it is equal to ρA. Where ρ is mass per unit volume (Kg/m^3).

The work done on the beam is given by :

$$W = F * w \qquad A5$$

where F is a force which is generated by the load which is acting on the beam.

Applying Hamilton's principle to the Lagrangian L=T-U+W of the system, namely:

$$\delta \int_{t_1}^{t_2} L dt = 0 \qquad A6$$

one can extract the following differential equations. In the w direction,

$$EI\,\partial^4 w/\partial x^4 - S\,\partial^2 w/\partial x^2 - (\partial/\partial x)\left\{EA\left[\partial u/\partial x + 1/2(\partial w/\partial x)^2\right](\partial w/\partial x)\right\}$$
$$+\rho A\,\partial^2 w/\partial t^2 - F(x,t) + C\,\partial w/\partial t = 0. \qquad A7$$

In the u direction,

$$\rho A\,\partial^2 u/\partial t^2 - (\partial/\partial x)\left\{EA\left[\partial u/\partial x + 1/2(\partial w/\partial x)^2\right]\right\} = 0. \qquad A8$$

where $C\partial w/\partial t$ is a equivalent viscous term added to the equation to account for the damping forces . F(x,t) is equivalent to the static and dynamic loads which are produced by the moving vehicle and it is equal to :

$$\left[(m_s + m_u)g - m_s\ddot{y}_3 - m_u\ddot{y}_2\right]\delta(x - vt) \qquad A9$$

Thus, the beam motion is governed by the differential equations A7 and A8.

2. For the vehicle motion :

By considering the body mass (M_s) of the car, the following forces will act on it :

1. The attraction force of gravity.

$$F_1 = m * g \qquad A10$$

where g is the acceleration of gravity .

2. The spring force (k_s) exerted by the spring when the spring is stretched . This force is proportional to the stretch

$$F = k * s \qquad \text{Hooke's law}$$

where

s : is the stretch

k : is the spring modulus

When the body is at rest we describe its position as the static equilibrium position. Clearly in this position the spring is stretched by an amount s_0 such that the resultant of the corresponding spring force and the gravitational force A10 is zero, that is

$$k * s_0 = m * g \qquad \text{A11}$$

Let y=y(t) denote the displacement of the body from the static equilibrium position . From Hooke's law it follows that the spring force corresponding to a dislpacement y is :

$$F_2 = -k * s_0 - k * y \qquad \text{A12}$$

3. If we connect the mass to a dashpot, then we have to take the corresponding viscous damping into account. The corresponding damping force has the direction opposite to the instantoneous motion , and it is proportional to the velocity $\dot{y} = dy/dt$ of the body . Thus the damping force is of the form :

$$F_3 = -C_s * \dot{y} \qquad \text{A13}$$

The resultant of the forces acting on the body (M_s) is equal to :

$$F_1 + F_2 + F_3 = mg - k * s_0 - k_s y - C_s \dot{y} \qquad \text{A14}$$

and because of equation A11 , equation A14 becomes

$$F_1 + F_2 + F_3 = -k_s y - C_s \dot{y} \qquad \text{A15}$$

Hence, by Newton's second law

$$M_s \ddot{y} = -k_s y - C_s \dot{y} \qquad \text{A16}$$

When the system in motion , the net change in spring length y equal to $(y_3 - y_2)$.

Equation A16 becomes as

$$M_s \ddot{y}_3 + k_s (y_3 - y_2) + C_s (\dot{y}_3 - \dot{y}_2) = 0. \qquad \text{A17}$$

By considering the forces which is acting on the mass of the wheel (M_u) we will have,

$$M_u\ddot{y}_2 + k_s(y_2 - y_3) + C_s(\dot{y}_2 - \dot{y}_3) + k_p(y_2 - y_1) = 0. \qquad A18$$

where y_1 is the transverse deflection of the beam at the point just under the movung load. Rearranging equation (A17) , we have

$$M_s\ddot{y}_3 = k_s(y_2 - y_3) + C_s(\dot{y}_2 - \dot{y}_3) \qquad A19$$

Substituting equation (A19) into equation (A18) we will have

$$M_u\ddot{y}_2 + M_s\ddot{y}_3 + k_p(y_2 - y_1) = 0. \qquad A20$$

Thus , the vehicle motion is governed by equations (A17) and (A20).

APPENDIX II

STEP- BY- STEP INTEGRATION BY THE NEWMARK METHOD [23,26,30]

Initial calculations proceed as follows.

I. form the linear stiffness matrix [K] , the mass matrix [M] and the damping matrix [C].Calculate the following constants (for tol. $\leq .001$, $\alpha \geq .5, \beta \geq .25(.5+\alpha)$)

$$A_0 = 1/\beta(dt)^2, \qquad A_1 = \alpha/\beta(dt), \qquad A_2 = 1/\beta(dt),$$

$$A_3 = 1/(2.\beta)-1., \qquad A_4 = (\alpha/\beta)-1., \qquad A_5 = dt[(\alpha/\beta)-2.]/2.,$$

$$A_9 = dt(1.-\alpha), \qquad A_{10} = \alpha dt,$$

II. Initialize $\{\Re\}_0, \{\dot{\Re}\}_0$ and $\{\ddot{\Re}\}_0$

III. From the effective linear stiffness matrix

$$[\hat{K}] = [K] + A_0[M] + A_1[C]$$

where [K]= [KL] +[KG]

The step-by-step integration the proceeds as follows.

I. The updated stiffness matrix [KG] for non-linear effects is to be formed.

II. Form the effective load vector

$$\{\hat{F}\}^{t+dt} = \{F\}^{t+dt} + [M]\left(A_2\{\dot{\Re}\}^t + A_3\{\ddot{\Re}\}^t\right) + [C]\left(A_4\{\dot{\Re}\}^t + A_5\{\ddot{\Re}\}^t\right) - \{R\}^t,$$

III. Solve for the displacement increments.

$$[\hat{K}]\{d\Re\} = \{F\}^{t+dt},$$

IV. Iterate for equilibruim if required.

$$\{d\Re\}^i = \{d\Re\}, \qquad i = 0.0$$

1. i=i+1

2. Calculate $(i-1)^{th}$ approximation accelerations, velocities and displacement

$$\{\ddot{\Re}\}^{i-1}_{t+dt} = A_0\{d\Re\}^{i-1} - A_2\{\dot{\Re}\}_t - A_3\{\ddot{\Re}\}_t$$

$$\{\dot{\Re}\}^{i-1}_{t+dt} = A_1\{d\Re\}^{i-1} - A_4\{\dot{\Re}\}_t - A_5\{\ddot{\Re}\}_t$$

$$\{\Re\}^{i-1}_{t+dt} = \{d\Re\}^{i-1} + \{\Re\}_t$$

3. Calculate $(i-1)^{th}$ effective out-of-balance loads

$$\left\{\hat{F}\right\}_{t+dt}^{i-1}=\{F\}_{t+dt}-[M]\left\{\ddot{\Re}\right\}_{t+dt}^{i-1}-[C]\left\{\dot{\Re}\right\}_{t+dt}^{i-1}-\{R\}_{t+dt}^{i-1},$$

4. Calculate $(i-1)^{th}$ stiffness matrix $\left[\hat{K}\right]_t^{i-1}$.

5. Solve for i^{th} correction to displacement increments :

$$[K]_t^{i-1}\{\delta\Re\}^i=\{F\}_{t+dt}^{i-1}$$

6. Calculate new displacement increments :

$$\{\Delta\Re\}^i=\{d\Re\}^{i-1}+\{\delta\Re\}^i$$

7. Ineration convergence if,

$$\left\|\{\delta\Re\}^i\right\|/\left\|\{\Re\}_{t+dt}^{i-1}+\{\delta\Re\}^i\right\|\leq \text{tol.}$$

if convergence then $\{d\Re\}=\{\Delta\Re\}^i$ and go to V ; if no convergence go to 1;

otherwise restart using a smaller time step size.

V. Calculate new accelerations, velocities and displacements:

$$\left\{\ddot{\Re}\right\}^{t+dt}=A_0\{d\Re\}-A_2\left\{\dot{\Re}\right\}^t-A_3\left\{\ddot{\Re}\right\}^t$$

$$\left\{\dot{\Re}\right\}^{t+dt}=\left\{\dot{\Re}\right\}^t+A_9\left\{\ddot{\Re}\right\}^t+A_{10}\left\{\ddot{\Re}\right\}^{t+dt}$$

$$\{\Re\}^{t+dt}=\{\Re\}+\{d\Re\}.$$

APPENDIX III

COMPUTER PROGRAM

```
INTEGER  N,I,K,IPVT(10),IPVTMT,NLESS1,IPLUS1,K2,K3,KCOL,JCOL
INTEGER  JROW,TMPVT,MM,T,NLE2,IP(10),TMP,IPL2,IPV,DK,K1,I1,I2,J2
INTEGER  I3,J3,I4,K4,A,B,W,TF,IVT(10),NESS1,ILUS1,IVTMT,NOF,NDIM
INTEGER  TPVT,
REAL     H,F0(10,10),PI,L,MT(10,10),AC(10,10),F6(10,10),C,V,AT(10,10)
REAL     BE,KL(10,10),MA(10,10),KA(10,10),A0,A1,A2,A3,A4,A5,A6,A7
REAL     A8,A9,A10,RA(10),DX,M3,WN,WN1,WN2,M4,M2,M6,DT,EL
REAL     TR(10),TE(10),SM(10,10),SC(10,10),SK(10,10),F1(10,10)
REAL     F2(10,10),SUM1,SUM2,XC,F(10),TR1(10),DIS2(10),SUM6(10,10)
REAL     DIS(10),VEL(10),ACC(10),F4(10),M1,DET,SUM,M8(10,10),SAVE
REAL     RATIO,VALUE,X(10),L1,WR1(10),TA(10,10),Y1(10),DIS1(10)
REAL     VEL1(10),ACC1(10),TOT1,CA(10,10),F3(10,10),WR(10),F5(10)
REAL     M9(10),M7(10),F52(10,10),SVE,DKK,D,DISL(10),VELL(10)
REAL     ACCL(10),KG(10,10),F50(10,10),DET1,X1(10),SUM3,DISL1(10)
REAL     SUM7(10,10),F7(10,10),WR5(10),WR6(10),AE5(10),AR5(10),
REAL     ACCL1(10),SUM10(10),VELL1(10),DISL2(10),Y2(10),Y3(10)
REAL     SUM20(10,10),R(10),SUM11(10),SUM12(10,10),SUM13(10,10)
REAL     SUM21(10,10),F10,F11(10),F12,F13,F14,F15,F16(10,10),F17
REAL     SUM22(10,10),F18(10),F19,F20,F21,F22,F23(10,10),F25(10),F24
REAL     SUM99(10,10),F26,F27,F28,F29,F30(10,10),F31,F51(10,10),EF
REAL     AM1,IM,F40,F41,F42,MV,QS,GRV,SUM5(10,10),ELF,CS,SR1
REAL     SUM100,SUM200,SUM300,DIS4(10),SR,DI(10),SU,SU1,NIT(10)
REAL     NIT1(10),NIT2(10),NIT3(10),DIS3(10),SZ,AVE,ACC2(10),DET5
REAL     VEL2(10),DIS5(10),SUMM,XX(10),F59(10,10),MU,MS,KS,KP
WRITE(*,*)' ENTER STARTING POINT OF THE BEAM    A          (M)'
READ(*,*)  A
WRITE(*,*)' ENTER FINAL POINT OF THE BEAM       B     (M)'
READ (*,*) B
WRITE (*,*)'ENTER NO. OF MODES                  N            '
READ (*,*) N
WRITE (*,*) 'ENTER YOUNG'S MODULUS              E       N/M^2 '
READ(*,673) E
WRITE(*,*)'ENTER MASS PER UNIT VOLUME           ROH     KG/M^3'
READ(*,673)  ROH
WRITE(*,*)'ENTER THICKNESS OF THE BEAM          TH         (M)'
READ (*,*)  TH
WRITE(*,*)'ENTER WIDTH OF THE BEAM              WI      (M)'
READ (*,*) WI
```

```
      WRITE (*,*)'ENTER MOMENT OF INERTIA OF AREA          MO   M4'
      READ(*,*)  MO
      WRITE(*,*)'ENTER SPRING MASS OF THE MOVING LOAD    MS   KG'
      READ(*,673), MS
      WRITE(*,*)'ENTER UNSPRING MASS OF THE MOV. LOAD   MU   KG'
      READ(*,*)  MU
      WRITE(*,*)'ENTER SPRING CONSTANT                 KS      N/M   '
      READ(*,673)  KS
      WRITE(*,*)' ENTER SPRING CONSTANT      KP      N/M'
      READ(*,*)  KP
      WRITE(*,*)'ENTER VELOCITY OF THE VEHICLE        V       M/SEC  '
      READ  (*,*)  V
      WRITE(*,*)'ENTER DAMPING COEFF.  OF THE VEHICLE    CS    KG/S '
      READ(*,673) CS
673   FORMAT(E10.15)
      L=B-A
      ELF=1.0
      C=0.0
      PI=22./7.
      G=9.80665
      AR=TH*WI
      MOO=MO/(L**4)
      VE=(SQRT(ROH/E))*V
      ROF=(SQRT(MO/AR))/L
      GRV=ROH*L*G/E
      MV=(MS+MU)/ROH*(L**3)
      FTT=1.0/VE
      FT=FTT+200
      M3=MU/(MU+MS)
      M6=MV*GRV
      WN=SQRT((KS*(L**2)*ROH)/(E*(MU+MS)))
      WN1=SQRT((KP*(L**2)*ROH)/(E*(MS+MU)))
      WN2=CS/(2.0*SQRT(KS*(MU+MS)))
      NDIM=10
      NOF=N+2
      DT=2.
      EL=.5
      BE=.25
      A0=1./(BE*(DT**2))
      A1=EL/(BE*DT)
      A2=1./(BE*DT)
      A3=(1./2.*BE))-1.
      A4=(EL/BE)-1
      A5=(DT/2.)*((EL/BE)-2.)
      A9=DT*(1.-EL)
      A10=EL*DT

C     COMPUTING THE TRANSVERSE MASS MATRIX       [MT]
C     AXIAL STIFFNESS MATRIX                     [KA]
```

```
C       LINEAR STIFFNESS MATRIX                         [KL]
        DO 549 I=1,N
        DO 549 J=1,N
        F2(I,J)=0.0
        F52(I,J)=0.0
        F6(I,J)=0.0
549     CONTINUE
        DO 1 K1=A,B
        DO 2 I2=1,N
        DO 2 J2=1,N
        SUM5(I2,J2)=SIN((I2*PI*K1)/L)*SIN((J2*PI*K1)/L)
        SUM5(I2,J2)=SUM5(I2,J2)*(-ELF*ABS((K1/L)-.5)+1.)
        SUM6(I2,J2)=(I2*PI**2)*SIN(I2*PI*K1/L)*(J2*PI**2)*SIN(J2*PI*K1/L)
        SUM6(I2,J2)=SUM6(I2,J2)*((ROG)**2)*(-ELF*ABS((K1/L)-.5)+1.)
        SUM7(I2,J2)=(I2*PI)*COS(I2*PI*K1/L)*(J2*PI)*COS(J2*PI*K1/L)
        SUM7(I2,J2)=SUM7(I2,J2)*(-ELF*ABS((K1/L)-.5)+1.)
2       CONTINUE
        IF(K1.EQ.A) THEN
        DO 500 I=1,N
        DO 500 J=1,N
        F0(I,J)=SUM5(I,J)
        F50(I,J)=SUM6(I,J)
        F3(I,J)=SUM7(I,J)
500     CONTINUE
        ENDIF
        IF(K1.EQ.B) THEN
        DO 501 I=1,N
        DO 501 J=1,N
        F1(I,J)=SUM5(I,J)
        F51(I,J)=SUM6(I,J)
        F7(I,J)=SUM7(I,J)
501     CONTINUE
        ENDIF
        IF(K1.GT.A.AND.K1.LT.B) THEN
        DO 502 I=1,N
        DO 502 J=1,N
        F2(I,J)=F2(I,J)+SUM5(I,J)
        F52(I,J)=F52(I,J)+SUM6(I,J)
        F6(I,J)=F6(I,J)+SUM7(I,J)
502     CONTINUE
        ENDIF
1       CONTINUE
        DO 504 I=1,N
        DO 504 J=1,N
        MT(I,J)=(DX/2.)*(F0(I,J)+F1(I,J)+2.*F2(I,J))
        KL(I,J)=(DX/2.)*(F50(I,J)+F51(I,J)+2.*F52(I,J))
        KA(I,J)=(DX/2.)*(F3(I,J)+F7(I,J)+2.*F6(I,J))
504     CONTINUE
```

```
C     AXIAL MASS MATRIX   [MA]
      DO 105 I=1,N
      DO 105 J=1,N
      MA(I,J)=MT(I,J)
      CA(I,J)=0.0
      AC(I,J)=C*MT(I,J)
105   CONTINUE
      DO 89 I-1,N
      DO 89 J=1,N
      TA(I,J)=KA(I,J)+A0*MA(I,J)+A1*CA(I,J)
89    CONTINUE
      DO 7 T=0,INT(FT),2
      IF(T.EQ.0.0) THEN
      DO 22 I=1,N+2
      DIS(I)=0.0
      VEL(I)=0.0
      ACC(I)=0.0
      DISL(I)=0.0
      VELL(I)=0.0
      ACCL(I)=0.0
22    CONTINUE
      ENDIF
C     COMPUTING THE GEOMETRIC STIFFNESS MATRIX   [KG]
      DO 19 I=1,N
      DO 19 J=1,N
      SUM22(I,J)=0.0
19    CONTINUE
      DO 8 K1=A,B,1
      F10=0.0
      DO 9 K=1,N
      F10=F10+DIS(K)*((K*PI)*COS(K*PI*K1/L))
      DO 10 I1=1,N
      F11(I1)=F10*(J1*PI)*COS(J1*PI*K1/L)
      F12=0.0
      DO 15 K2=1,N
      F12=F12+F11(K2)+DIS(K2)
      F13=.5*F12
      F14=0.0
      DO 16 K3=1,N
      F14=F14+((K3*PI)*COS(K3*PI*K1/L)*DIS1(K3)
      F15=F14+F13
      DO 956 I=1,N
      DO 956 J=1,N
      F16(I,J)=0.0
956   CONTINUE
      DO 17 I=1,N
      DO 17 J=1,N
      F16(I,J)=F16(I,J)+(I*PI)*COS(I*PI*K1/L)*(J*PI)*COS(J*PI*K1/L)
      F16(I,J)=F16(I,J)*(-ELF*ABS((K1/L)-.5)+1.)
```

```
      F16(I,J)=F16(I,J)*F15
17    CONTINUE
16    CONTINUE
15    CONTINUE
10    CONTINUE
9     CONTINUE
      IF(K1.EQ.A) THEN
      DO 11 I=1,N
      DO 11 J=1,N
      SUM20(I,J)=F16(I,J)
11    CONTINUE
      ENDIF
      IF(K1.EQ.B) THEN
      DO 12 I=1,N
      DO 12 J=1,N
      SUM21(I,J)=F16(I,J)
12    CONTINUE
      ENDIF
      IF(K1.GT.A.AND.K1.LT.B) THEN
      DO 13 I=1,N
      DO 13 J=1,N
      SUM22(I,J)=SUM22(I,J)+F16(I,J)
13    CONTINUE
      ENDIF
8     CONTINUE
      DO 767 I=1.N
      DO 767 J=1,N
      KG(I,J)=(DX/2.)*(SUM20(I,J)+SUM21(I,J)+2.*SUM22(I,J))
767   CONTINUE
14    FORMAT(5(E15.3))
C     FORMING THE COUPLED EQUATION
      DO 30 I=1,N+2
      DO 30 J=1,N+2
      SM(I,J)=0.0
      SC(I,J)=0.0
      SK(I,J)=0.0
30    CONTINUE
      IF(T.GR.FTT) THEN
      MV=0.0
      ENDIF
      DO 32 I=1,N
      DO 32 J=N+1,N+1
      SM(I,J)=MV*M3*SIN(I*PI*VE*T)
32    CONTINUE
      DO 33 I=1,N
      DO 33 J=N+2,N+2
      SM(I,J)=MV*(1-M3)*SIN(I*PI*VE*T)
33    CONTINUE
      SM(N+1,N+1)=M3
```

```
      SM(N+1,N+2)=(1-M3)
      SM(N+2,N+2)=1-M3
      SC(N+2,N+1)=-(2.*WN2*WN))
      SC(N+2,N+2)=+(2.*(WN2*WN))
      SK(N+1,N+1)=WN1**2
      SK(N+2,N+1)=-(WN**2)
      SK(N+2,N+2)=WN**2
      DO 34 I=N+1,N+1
      DO 34 J=1,N
      SK(I,J)=-(WN1**2)*(SIN(J*PI*VE*T))
34    CONTINUE
      DO 35 I=1,N
      DO 35 J=1,N
      SM(I,J)=MT(I,J)
      SC(I,J)=AC(I,J)
      SK(I,J)=KL(I,J)
35    CONTINUE
C     COMPUTING THE TOTAL MATRIX [AT]
      DO 41 I=1,N+2
      DO 41 J=1,N+2
      AT(I,J)=SK(I,J)+A0*SM(I,J)+A1*SC(I,J)
41    CONTINUE
C     COMPUTING THE TOTAL FORCE
      DO 43 I=1,N+2
      F4(I)=0.0
43    CONTINUE
      IF(T.GT.FTT) THEN
      M6=0.0
      ENDIF
      DO 44 I=1,N
      F(I)=M6*SIN(I*PI*VE*(T+DT))
44    CONTINUE
      DO 45 I=1,N
      F4(i)=F(i)
45    CONTINUE
      DO 46 I=1,N+2
      TR(I)=A2*VEL(I)+A3*ACC(I)
      TE(I)=A4*VEL(I)+A5*ACC(I)
46    CONTINUE
      DO I=1,N+2
      WR(I)=0.0
      DO 47 K=1,N+2
      WR(I)=WR(I)+SM(I,K)*TR(K)
47    CONTINUE
      DO 48 I=N+2
      WR1(I)=0.0
      DO 48 K=1,N+2
      WR1(I)=WR1(I)+SC(I,K)*TE(K)
48    CONTINUE
```

```
      DO 107 I=1,N+2
      R(I)=0.0
      DO 107 K=1,N+2
      R(I)=R(I)+SK(I,K)*DIS(K)
107   CONTINUE
      DO 49 I=1,N+2
      TR1(I)=F4(I)+WR(I)+WR1(I)-R(I)
49    CONTINUE
C     THIS PART SOLVES THE EQUATION BY GAUSSIAN ELIMINATION
C     WITH PARTIAL PIVOTING AND BACK SUBSTITUTION
      DET=1.0
      NLESS1=NOF-1.
      DO 51 I=1,NOF
      IPVT(I)=I
51    CONTINUE
      DO 52 I=1,NLESS1
      IPLUS1=I+1
      IPVTMT=I
      DO 53 J=IPLUSU1,NOF
      IF(ABS(AT(IPVTMT,I)).LT.ABS(AT(J,I))) IPVTMT=J
53    CONTINUE
      IF(IPVTMT.NE.I0 THEN
      TMPVT=IPVT(I)
      IPVT(I)=IPVT(IPVTMT)
      DO 820 JCOL=I,NOF
      SAVE=AT(I,JCOL)
      AT(I,JCOL)=AT(IPVTMT,JCOL)
      AT(IPVTMT,JCOL)=SAVE
820   CONTINUE
      IPVT(IPVTMT)=TMPVT
      DET=-DET
      ENDIF
      DO 821 JROW=IPLUS1,NOF
      IF(AT(JROW,I).NE.0.0) THEN
      AT(JROW,I)=AT(JROW,I)/AT(I,I)
      DO 822 KCOL=IPLUS1,NOF
      AT(JROW,KCOL)=AT(JROW,KCOL)-AT(JROW,I)*AT(I,KCOL)
822   CONTINUE
      ENDIF
821   CONTINUE
52    CONTINUE
      DO 54 I=1,NOF
      DET=DET*AT(I,I)
54    CONTINUE
      DO 55 I=1,NOF
      X(I)=TR1(IPVT (I))
55    CONTINUE
      DO 56 IROW=2,NOF
      SUM=X(IROW)
```

```
      DO 57 JCOL=1,(IROW-1)
      SUM=SUM-AT(IROW,JCOL)*X(JCOL)
57    CONTINUE
      X(IROW)=SUM
56    CONTINUE
      DIS1(NOF)=X(NOF)/AT(NOF,NOF)
      DO 58 IROW=(NOF-1)1,-1
      SUM=X(IROW
      DO 59 JCOL=(IROW+1),NOF
      SUM=SUM-AT(IROW,JCOL)*DIS1(JCOL)
59    CONTINUE
      DIS1(IROW)=SUM/AT(IROW,IROW)
58    CONTINUE
846   DO 800 I=1,N+2
      ACC1(I)=A0*DIS1(i)-A2*VEL(i)-A3*ACC(i)
      VEL1(i)=A1*DIS1(i)-A4*VEL(i)-A5*ACC(i)
      DIS2(i)=DIS1(i)+DIS(i)
800   CONTINUE
      DO 801 I=1,N+2
      NIT(i)=0.0
      NIT1(i)=0.0
      NIT2(i)=0.0
      DO 801 K=1,N+2
      NIT(i)=NIT(i)+SM(i,k)*ACC1(k)
      NIT1(i)=NIT1(i)+SC(i,k)*VEL1(k)
      NIT2(i)=NIT2(i)+SK(i,k)*DIS2(k)
801   CONTINUE
      DO 802 I=1,N+2
      NIT3(i)=F4(i)-NIT(i)-NIT1(i)-NIT2(i)
802   CONTINUE
      DET5=1.0
      NESS1=NOF-1
      DO 803 I=1,NOF
      IVT(i)=I
803   CONTINUE
      DO 804 I=1,NESS1
      ILUS=I+1
      IVTMT=I
      DO 805 J=ILUS1,NOF
      IF(ABS(AT(IVTMT,I)).LT.ABS(AT(J,I))) IVTMT=J
805   CONTINUE
      IF(IVTMT.NE.I) THEN
      TPVT=IVT(i)
      IVT(i)=IVT(IVTMT)
      DO 333 JCOL=1,NOF
      SVE=AT(I,JCOL)
      AT(I,JCOL)=AT(IVTMT,JCOL)
      AT(IVTMT,JCOL)=SVE
333   CONTINUE
```

```
      IVT(IVTMT)=TPVT
      DET5=-DET5
      ENDIF
      DO 334 JROW=ILUS1,NOF
      IF(AT(JROW,I).NE.0.0) THEN
      AT(JROW,I)=AT(JROW,I)/AT(I,I)
      DO 335 KCOL=ILUS1,NOF
      AT(JROW,KCOL)=AT(JROW,KCOL)-AT(JROW,I)*AT(I,KCOL)
335   CONTINUE
      ENDIF
334   CONTINUE
804   CONTINUE
      DO 806 I=1,NOF
      DET5=DET5*AT(I,I)
806   CONTINUE
      DO 807 I=1,NOF
      XX(i)=NIT3(IVT(i))
807   CONTINUE
      DO 808 IROW=2,NOF
      SUMM=XX(i)
      DO 809 JCOL=1,(IROW-1)
      SUMM=SUMM-AT(IROW,JCOL)*XX(JCOL)
809   CONTINUE
      XX(IROW)=SUMM
808   CONTINUE
      DIS3(NOF)=XX(NOF)/AT(NOF,NOF)
      DO 810 IROW=(NOF-1),1,-1
      SUMM=XX(IROW)
      DO 811 JCOL=(IROW+1),NOF
      SUMM=SUMM-AT(IROW,JCOL)*DIS(JCOL)
811   CONTINUE
      DIS3(IROW)=SUMM/AT(IROW,IROW)
810   CONTINUE
      DO 840 I=1,N+2
      DIS4(i)=DIS(i)+DIS3(i)
840   CONTINUE
      SR=0.0
      DO 841 K=1,N+2
      SR=SR+DIS3(k)*DIS3(k)
841   CONTINUE
      SR1=SQRT(SR)
      DO 842 I=1,N+2
      DI(i)=DIS2(i)+DIS3(i)
842   CONTINUE
      SU=0.0
      DO 843 K=1,N+2
      SU=SU+DI(k)*DI(k)
843   CONTINUE
      SU1=SQRT(SU)
```

```
        SZ=SR1/SU1
        IF(SZ.GT..001) GOTO 844
        IF(SZ.LT..001) GOTO 888
844     DO 845 I=1,N+2
845     DIS1(i)=DIS4(i)
        GOTO 846
888     DO 62 I=1,N+2
        ACC2(i)=A0*DIS4(i)-A2*VEL(i)-A3*ACC(i)
        VEL2(i)=VEL(i)+A9*ACC(i)+A10*ACC2(i)
        DIS5(i)=DIS(i)+DIS4(i)
62      CONTINUE
        DO 773 I=1,N
        Y3(i)=0.0
773     CONTINUE
        DO 600 K1=A,B
        DO 64 I=1,N
        DO 64 J=1,N
        SUM10(i)=(I*PI)*COS(I*PI*K1/L)*(-ELF*ABS((K1/L)-.5)+1)
        DO 883 I1=1,N
        DO 883 J1=1,N
        SUM99(I1,J1)=0.0
883     CONTINUE
        DO 884 I1=1,N
        DO 884 J1=1,N
        SUM99(I1,J1)=SUM99(I1,J1)+SUM10(I1)*DIS5(J1)
        DO 605 I2=1,N
        SUM11(I2)= 0.0
        DO 605 K=1,N
        SUM11(I2)=SUM11(I2)+SUM99(I2,K)*((K,PI)*COS(K*PI*K1/L))
        DO 885 I3=1,N
        DO 885 J3=1,N
        SUM12(I3,J3)=0.0
885     CONTINUE
        DO 606 I3=1,N
        DO 606 J3=1,N
        SUM12(I3,J3)=SUM12(I3,J3)+SUM11(I3)*((J3*PI)*COS(J3*PI*K1/L))
        DO 607 I4=1,N
        SUM13(I4)=0.0
        DO 607 K4=1,N
        SUM13(I4)=SUM13(I4)+SUM12(I4,K4)*DIS5(K4)
607     CONTINUE
606     CONTINUE
605     CONTINUE
884     CONTINUE
64      CONTINUE
        IF(K1.EQ.A) THEN
        DO 601 I=1,N
        Y1(i)=SUM13(i)
601     CONTINUE
```

```
      ENDIF
      IF(K1.EQ.B) THEN
      DO 602 I=1,N
      Y2(i)=SUM13(i)
602   CONTINUE
      ENDIF
      IF(K1.GT.A.AND.K1.LT.B) THEN
      DO 603 I=1,N
      Y3(i)=Y3(i)+SUM13(i)
603   CONTINUE
      ENDIF
600   CONTINUE
      DO 608 I=1,N
      M9(i)=-(.5*DX/2.)*(Y1(i)+Y2(i)+2.*Y3(i))
608   CONTINUE
      DO 84 I=1,N
      AR5(i)=A2*VELL(i)+A3*ACCL(i)
      AE5(i)=A4*VELL(i)+A5*ACCL(i)
84    CONTINUE
      DO 85 I=1,N
      WR5(i)=0.0
      DO 85 K=1,N
      WR5(i)=WR5(i)+MA(I,K)*AR5(k)
85    CONTINUE
      DO 86 I=1,N
      WR6(i)=0.0
      DO 86 K=1,N
      WR6(i)=WR6(i)+CA(I,K)*AE5(k)
86    CONTINUE
      DO 108 I=1,N
      RA(i)=0.0
      DO 108 K=1,N
      RA(i)=RA(i)+KA(I,K)*DISL(K)
108   CONTINUE
      DO 87 I=1,N
      M7(i)=M9(i)+WR5(i)+WR6(i)-RA(i)
87    CONTINUE
      DET1=1.0
      NLE2=N-1
      DO 91 I=1,N
      IP(i)=1
91    CONTINUE
      DO 92 I=1,NLE2
      IPL2=I+1
      IPV=I
      DO 93 J=IPL2,N
      IF(ABS(TA(IPV,I)).LT.ABS(TA(J,I)))  IPV=J
93    CONTINUE
      IF(IPV.NE.I) THEN
```

```
      TMP=IP(i)
      IP(i)=IP(IPV)
      DO 890 JCOL=1,N
      AVE=TA(I,JCOL)
      TA(I,JCOL)=TA(IPV,JCOL)
      TA(IPV,JCOL)=AVE
890   CONTINUE
      IP(IPV)=TMP
      DET1=-DET1
      ENDIF
      DO 891 JROW=IPL2,N
      IF(TA(JROW,I).NE.0.0) THEN
      TA(JROW,I)=TA(JROW,I)/TA(I,I)
      DO 892 KCOL=IPL2,N
      TA(JROW,KCOL)=TA(JROW,KCOL)-TA(JROW,I)*TA(I,KCOL)
892   CONTINUE
      ENDIF
891   CONTINUE
92    CONTINUE
      DO 94 I=1,N
      DET1=DET1*TA(I,I)
94    CONTINUE
      DO 95 I=1,N
      X1(i)=M7(IP(i))
95    CONTINUE
      DO 96 IROW=2,N
      SUM3=X1(IROW)
      DO 97 JCOL =1,(IROW-1)
      SUM3=SUM3-TA(IROW,JCOL)*X1(JCOL)
97    CONTINUE
      X1(IROW)=SUM3
96    CONTINUE
      DISL1(N)=X1(N)/TA(N,N)
      DO 98 IROW=(N-1),1,-1
      SUM3=X1(IROW)
      DO 99 JCOL =(IROW+1),N
      SUM3=SUM3-TA(IROW,JCOL)*DISL1(JCOL)
99    CONTINUE
      DISL1(IROW)=SUM3/TA(IROW,IROW)
98    CONTINUE
      DO 100 I=1,N
      ACCL1(i)=A0*DISL1(i)-A2*VELL(i)-A3*ACCL(i)
      VELL1(i)=VELL(i)+A9*ACCL(i)+A10*ACCL1(i)
      DISL2(i)=DISL(i)+DISL1(i)
100   CONTINUE
      XC=0.0
      DO 102 K=1,N
102   XC=XC+DIS(k)*SIN(K*PI*.5)
      WRITE(*,109) T,XC
```

```
109   FORMAT(,'  ',I4,'     ',E15.3)
      DO 103 I=1,N+2
      DIS(i)=DIS5(i)
      VEL(i)=VEL2(i)
      ACC(i)=ACC2(i)
103   CONTINUE
      DO 540 I=1,N
      DISL(i)=DISL2(i)
      VELL(i)=VELL1(i)
      ACCL(i)=ACCL1(i)
540   CONTINUE
7     CONTINUE
      STOP
      END
```

الملخــص

الاستجابة اللاخطية للجيزان المعرضة لحمل متحرك

إعداد

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إشـــراف

الدكتور مازن القيسي

من المعروف أن الجيزان تمثل أحد أهم العناصر الانشائية في التصميم الهندسي والإنشائي، ولا يوجد تصميم يخلو من مشكلات الجيزان ، إذ تدخل في تصميم الجسور المعرضة لتلك الأحمال المتحركة الناتجة عن حركة السير . ويكون المطلوب تجنب الانحناءات الديناميية الاضافية . وذلك – ليس فقط من أجل اعتبارات السلامة – بل لضمان سلامة المشاة وراحتهم وإتاحة القيادة الآمنة للسيارات أيضا . فالطرق التحليلية والعددية للتنبؤ بالانحناءات الدينامية تستلزم ذلك في تصميم الجسور .

وتمثل هذه الدراسة تحليل الانحناءات الدينامية للجيزان التي تتعرض للأحمال المتحركة (مع تشكيل الجسور وفقا لنموذج الجيزان). وتتضمن هذه الدراسة أيضا التأثيرات اللالخطية الفراغية الحادثة مع اعتبار أن الجسر مرن ومعلق تعليقا بسيطا ، وبأطراف غير متحركة مع افتراض أن الحمل المتحرك هو ثنائي درجة التغير ويسيرعلى الجسر من طرف لآخر. وسوف يتم حساب الانحناءات الدينامية للجسر والعربة المتحركة بطريقة جالاركين ، مع افتراض أن الانحناءات الدينامية مجموعة من الاقترانات الزمنية مضروبة باقترانات تقريبية . وسوف يتم حساب الاقتران الزمني بالطرق العددية باستخدام طريقة نيومارك لحل المعادلات التفاضلية اللاخطية .

والمعروف أن الأحمال المتحركة على الجيزان تسبب انحناءات دينامية واجهادات أكثر من تلك التي يسببها التأثير الساكن للحمل نفسه ، لأنه لا يمكن إهمال التأثيرات اللاخطية في معادلات الحركة . لذلك ستتم دراسة ايجاد الانحناءات الدينامية للجيزان المعرضة للأحمال المتحركة باستخدام طريقة جالاركين ، ودراسة تأثير المضاءلة على الانحناءات الدينامية ، ودراسة تأثير سرعة الحمل ووزنه على الانحناءات الدينامية .